The 3rd OCAMI-IMAG Joint Conference on Differential Geometry

July 10 - 14, 2026

Venue: I-site Namba, Osaka, Japan

Abstracts

Plenary Speakers

July 10.

Miguel Sánchez (Universidad de Granada)

Semi-Riemannian Cheeger Gromov convergence

Abstract

Cheeger-Gromov convergence has not been systematically developed in the semi-Riemannian setting because of the lack of uniqueness of limits under the Riemannian hypothesis, as well as the difficulty in obtaining general existence theorems. Here, we will see that the notion of anchored convergence (slightly more restrictive than the usual Riemannian pointed convergence) yields satisfactory results for the uniqueness of limits under C^2-convergence. In the Lorentzian signature, this regularity is lowered, and Riemannian results on the existence of limits become available by using Cauchy temporal functions as (strongly restrictive) anchors.

Based on joint work with S. Burgos and J.L. Flores, arXiv:2508.15441.

Yoshinori Hashimoto (Osaka Metropolitan University)

Rigidity of isotropic harmonic maps from elliptic curves to complex projective spaces

Abstract

Eells-Wood defined isotropic harmonic maps, by means of a Frenet-Serret type construction out of holomorphic maps. They also proved that harmonic maps from an elliptic curve to a projective space of nonzero degree are all isotropic. Their work attracted renewed attention among physicists in the last few years due to its role in the condensed matter physics as the generalised Landau levels. The main result of this talk is their rigidity, in the sense that two isotropic harmonic maps inducing the same metric on the elliptic curve are related by the unitary action on the target projective space, up to translations. This is a joint work with Bruno Mera and Tomoki Ozawa.

Tomás Otero (Universität Münster)

Cohomogeneity one actions on symmetric spaces

Abstract

In this talk, I will report on recent developments on the classification of cohomogeneity-one actions on symmetric spaces. The focus will be on symmetric spaces of "mixed type" (i.e. those whose universal cover splits as a nontrivial product $\widetilde{M} = M_+ \times M_0 \times M_-$, with M_+ of compact type, M_0 a Euclidean space, and M_- of noncompact type).

In a recent preprint with Ivan Solonenko and Hiroshi Tamaru, we show that cohomogeneity-one actions split with respect to the aforementioned decomposition, with the only exception of a family of "diagonal" actions (which we parameterize explicitly). This fully reduces the classification problem for cohomogeneity-one actions to symmetric spaces of a single type (which is mostly well-understood).

Diego Alfonso Marín Muñoz (Universidad de Granada)

Geometry of f-extremal domains in the 2-sphere

Abstract

Given a Riemannian manifold (M, g) and a Lipschitz function f, we say that a domain inside M is an f-extremal domain if it supports a solution to a semilinear overdetermined elliptic problem with non-linearity f. It is a well known fact in the literature that the theory of the existence and rigidity of f-extremal domains is closely linked to the theory of constant mean curvature surfaces (CMC) in M.

In this talk, we will explore this connection for f-extremal domains in the two sphere S^2. In particular, using techniques from the theory of CMC-surfaces, we will prove that, under certain conditions on the function f and the topology of the domain, an f-extremal domain in S^2 must exhibit significant symmetry. This talk is based on joint work with my supervisor, José M. Espinar.

Fernán González Ibáñez (Universidad de Granada)

Comparison methods for semilinear elliptic problems on Riemannian manifolds with a Ricci lower bound

Abstract

we develop a comparison method for positive solutions of the semilinear Dirichlet problem $\Delta u + f(u) = 0$ on domains $\Omega \subset M^n$ of a Riemannian manifold (M^n, g) with a Ricci lower bound $\mathrm{Ric} g \geq (n-1)kg$. Assuming admissibility and structural conditions on f, we prove a sharp pointwise gradient comparison, with a rigid characterization of the equality case. As applications, we derive an explicit isoperimetric-type inequality and a quantitative hot-spot localization estimate under natural convexity assumptions

July 11.

Shoichi Fujimori (Hiroshima University)

Higher genus minimal surfaces with two ends

Abstract

We prove the existence of complete minimal surfaces, in Euclidean 3-space, of arbitrary positive genus and least total absolute curvature that have precisely two ends: a catenoidal end and an Enneper-type end.

This talk is based on a joint work with Rivu Bardhan, Indranil Biswas, and Pradip Kumar. https://arxiv.org/abs/2509.03925

Magdalena Rodríguez (Universidad de Granada)

Tensile minimal surfaces

Abstract

In architecture, minimal surfaces belong to the structural family of tensile surfaces. Tensile surfaces are lightweight, form-active structural membranes held in tension by cables, masts, or air pressure, creating durable and efficient shapes. The study of these surfaces saw significant progress in the 19th century, mainly thanks to the work of Frei Otro and his team.

In this talk we will introduce a method to produce new minimal surfaces bounded by asymptotic lines with contant curvature that provide tensile structures very efficient in architecture. These surfaces are solutions to certain thread problems as well. This is a joint work with Romane Boutillier and Laurent Hauswirth.

Sachiko Hamano (Kyoto Sangyo University)

Deformations of open Riemann surfaces: periods, Levi curvature, and rigidity

Abstract

Unlike the compact case, open Riemann surfaces lack a universally accepted notion of a period matrix. However, certain distinguished periods can be defined using the hydrodynamic differentials introduced by Masakazu Shiba.

In this talk, I will discuss the variational properties of these periods under complex deformations. After a brief review of hydrodynamic differentials and their associated period quantities, a second-order variational formula with respect to the deformation parameter will be presented. This formula explicitly expresses the variation in terms of the Levi curvature of the boundary and the Dirichlet norm of the parameter-derivative of the corresponding differential. As an application, the naturally associated span function is shown to be subharmonic under pseudoconvex variations of open Riemann surfaces. Moreover, the equality case yields a rigidity theorem characterizing trivial deformations.

July 12.

Franz Pedit (UMass Amherst)

Affine spheres, Calabi-Yau metrics, and Higgs bundles

Abstract

In this talk, I will explain how affine spheres—a special class of surfaces in affine differential geometry studied in the 1920s by Blaschke's school—play a role in the construction of Calabi–Yau metrics on special Lagrangian 3-torus fibrations. This construction relies on all three aspects of non-Abelian Hodge theory: Higgs bundles, surface group representations, and self-duality equations.

Charles Ouyang (Washington University in St. Louis)

Embedded special Legendrians in the 5-sphere

Abstract

Point singularities of Calabi-Yau 3-folds are modeled on special Legendrian cones in $\mathbb{C}^3$ whose link gives a special Legendrian in the 5-sphere. For tori, these have been well-studied, but in higher genus, there is a paucity of examples and only in select genera. We will discuss the construction of the first examples of embedded special Legendrian surfaces in the 5-sphere. This is joint work with Sebastian Heller and Franz Pedit.

July 13.

Pablo Mira (Universidad Politécnica de Cartagena)

Minimal surfaces and the Serrin problem

Abstract

A Serrin domain is a smooth domain of $\mathbb{R}^n$ where the Dirichlet solution to $\Delta u + 2 = 0$ has a constant Neumann value along its boundary. Bounded Serrin domains are balls after the famous theorem by J. Serrin, but there exist non-trivial unbounded ones. In this talk we will introduce integrable systems theory to the study of Serrin's problem in the plane, based on ideas from minimal and CMC surfaces. We will show that any annular domain $\Omega \subset \mathbb{R}^2$ where $\Delta u + 2 = 0$ can be solved with locally constant overdetermined boundary conditions is of finite type for the mKdV hierarchy. The same result holds for periodic bands. By analyzing a natural foliation structure we will find global moduli spaces of Serrin domains that are natural overdetermined analogues of well-known minimal and CMC surfaces, like Wente tori or Riemann's minimal examples. Joint work with Alberto Cerezo and Isabel Fernandez.

Yoshiki Jikumaru (Toyo University)

On the governing equations of membrane O surfaces

Abstract

It is known that a shell membrane in equilibrium where a constant purely normal load acts on the membrane, and where the principal curvature lines coincide with the principal stress lines, forms an integrable system called a membrane O surface (Rogers-Schief, 2003).

In this talk, we formulate the governing equations for membrane O surfaces of the 1st and 2nd kind, which are analogs to Guichard surfaces of the 1st and 2nd kind introduced by Calapso.

Furthermore, under this formulation, we show that membrane O surfaces are suitable subclasses of Demoulin's Ω surfaces, and that the Bäcklund transformation for membrane O surfaces preserves membrane O surfaces of the 1st and 2nd kind, respectively.

José M. Espinar (Universidad de Granada)

Genus two embedded minimal surfaces in S^3 with bidihedral symmetry

Abstract

The isometry group of the classical Lawson embedded minimal surface $\xi_{2,1} \subset S^3$ of genus 2 is isomorphic to the group $D_4 \times S_3$, where D_4 is the dihedral group of order 8 and S_3 the permutation group of order 6. $\mathrm{Iso}(\xi_{2,1})$ has a subgroup of index 3 isomorphic to the bidihedral group $D_{4h} = Z_2 \times D_4$. We will explain how to prove that $\xi_{2,1}$ is the unique closed embedded minimal surface of genus 2 in S^3 whose isometry group contains D_{4h}. This is a joint work with J. Pérez

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Chiharu Kosugi (Yamaguchi University)

Recent results on mathematical models for the elastic curve with the compressible stress function

Abstract

In this talk, we consider the initial and boundary value problem for the partial differential equation that describes the motion of an elastic material deforming by the bending of the curve and the strain. In our previous work, we have obtained several mathematical results for a beam equation derived from an energy functional in which the bending of the curve is represented by the Laplacian, by introducing a stress function with a singularity. We have proved the existence and uniqueness of weak and strong solutions. Moreover, we obtained the lower bound for the strain. This estimate guarantees that the elastic curve does not shrink to a single point. We emphasize that this result indicates advantage of our model. We remark that in our model, an unknown function is a vector-valued function and representing the position. For this reason, we need to treat the nonlinear strain function, and we adopt the singular stress function. For the construction of a model representing the characteristics of the elastic curve, we derive the initial

and boundary value problem from the free energy including the curvature. In this talk we would like to introduce our previous results and recent results and explain the advantage of the new model by comparing it with the behavior of the stationary solutions as the elastic coefficient tends to infinity. This is a joint work with Prof. Aiki from Japan Women's University, Japan.

Hirotaka Kiyohara (Osaka Kyoiku University)

Constructing timelike minimal surfaces with singularities in the three-dimensional Heisenberg group via theory of null curves

Abstract

A certain class of timelike minimal surfaces in the three-dimensional Heisenberg group can be characterized via Lorentzian harmonic maps into the de-Sitter 2-sphere. While singular points naturally occur on these surfaces, it is generally difficult to obtain explicit examples of such surfaces with specific singularities, as constructing harmonic maps with prescribed properties remains a challenge. In this talk, we introduce a duality between timelike constant mean curvature (CMC) surfaces in Minkowski space and timelike minimal surfaces in the Heisenberg group. Using this correspondence, we provide a method for constructing examples of timelike minimal surfaces with singularities, such as cuspidal cross caps and swallowtails, via the theory of framed null curves in Minkowski space. This talk is based on joint work with Shintaro Akamine.

July 14.

Víctor Sanmartín López (Universidade de Santiago de Compostela)

On some classes of hypersurfaces in symmetric spaces of noncompact type

Abstract

In submanifold theory, it is natural to begin by inspecting submanifolds with a high degree of symmetry, such as homogeneous hypersurfaces. They are always examples of isoparametric hypersurfaces with constant principal curvatures. In the first part of the talk, we will discuss the current state of the art concerning these three classes of hypersurfaces in symmetric spaces. We will then turn our attention to some of the most recent developments and advances in the investigation of isoparametric hypersurfaces.

Megumi Sano (Nara Women's University)

Weighted Trudinger-Moser inequalities in the subcritical Sobolev spaces and their applications

Abstract

We study boundedness, optimality and attainability of Trudinger-Moser type maximization problems in the radial and the subcritical homogeneous Sobolev spaces. Also, our inequality converges

to the original Trudinger-Moser inequality including optimal exponent and concentration limit. Finally, we consider an application of our inequality to PDEs with exponential nonlinearity.

Takeyuki Nagasawa (Saitama University)

A Möbius-Invariant Energy for Multi-Component Links via Relative Gauss Maps

Abstract

A Möbius-invariant energy for multi-component links was proposed by Freedman, He, and Wang in 1994. We introduce another Möbius-invariant energy for multi-component links. There is no natural map relating the two energies. However, the new energy admits a decomposition into Möbius-invariant components.

For two-component links, the Gauss map plays an important role in link theory. Indeed, the degree of the Gauss map is equal to the linking number, and the Gauss maps also appear in the Euler-Lagrange equation for the link energy. We define relative Gauss maps for multi-component links and explain how they appear in the Euler-Lagrange equation for the new energy.

Poster Presentations

Ángel Cidre Díaz (University of Santiago de Compostela)

Classification of minimal orbits in complex hyperbolic spaces

Abstract

Hyperbolic spaces are noncompact Riemannian manifolds with a high degree of symmetry, which geometrically means that they are symmetric spaces. In particular, symmetric spaces are also homogeneous, meaning that their group of isometries acts transitively on the space. This motivates the study of homogeneous submanifolds, which are the orbits of a subgroup of the isometry group of the ambient space. In particular, we focus on the case in which these orbits are minimal.

In real hyperbolic spaces, it is known that minimal orbits are totally geodesic, but in the case of the other hyperbolic spaces (i.e. complex hyperbolic spaces, quaternionic hyperbolic spaces and the Cayley hyperbolic plane) this no longer holds. In this poster, we present the main ideas of the classification of the minimal orbits of complex hyperbolic spaces.

Kaoru Iino (Yokohama National University)

Isometric m-type edges and symmetries

Abstract

Cuspidal edges are stable singularities appearing on wave fronts. As a class of singularities including cuspidal edges, m-type edge was introduced by Martins et al. In this study, for a given real analytic and admissible m-type edge, we classify the number of right-equivalence classes and congruence classes of m-type edges sharing the same image of the singular curve and the same first fundamental form. When m is even, our results generalize those of Honda et al. for generalized cuspidal edges. When m is odd, we obtain phenomena that do not appear in the case of generalized cuspidal edges. In particular, we show that a symmetry of the first fundamental form called a non-effective symmetry plays an essential role in the classification.

Masato Inagaki (The University of Osaka)

Spectral Approximation of Weighted Laplacians by Graph Laplacians under Lower Ricci Curvature Bounds

Abstract

This talk concerns spectral approximation by graph Laplacians constructed from data sampled from a Riemannian manifold. More precisely, we ask whether the eigenvalues and eigenvectors of neighborhood graph Laplacians approximate those of the corresponding weighted Laplacian on the underlying space.

I will present error estimates for this approximation problem that hold with high probability, where the graph Laplacians are constructed from independent and identically distributed samples

on closed Riemannian manifolds. A key feature of the result is that the estimates are obtained under relatively mild geometric assumptions: a lower Ricci curvature bound, an upper diameter bound, and a positive lower volume bound. In particular, the result does not require a two-sided sectional curvature bound or a positive injectivity radius.

This talk is based on https://arxiv.org/abs/2506.07427.

Hiroya Iwasaki (Osaka Metropolitan University)

Left-invariant Lorentzian steady Ricci solitons on nilpotent Lie groups

Abstract

Although Riemannian and pseudo-Riemannian geometries share many similarities, important differences also arise. In this study, we focus on homogeneous steady Ricci solitons to investigate the differences between the Lorentzian and Riemannian cases. In the Riemannian case, every homogeneous steady Ricci soliton is flat. In this presentation, we introduce a family of 2-step nilpotent Lie groups admitting non-Einstein left-invariant Lorentzian steady Ricci soliton metrics. This example reveals a difference between Riemannian and Lorentzian cases.

Shunsuke Kasao (Kanazawa University)

Bloch–Ros principle and its application to surface theory

Abstract

A duality exists between normal family theory and value distribution theory of meromorphic functions, which is called the Bloch principle. In this poster session, building on the works of Zalcman and Ros, we elucidate the trinity among normal family theory, value distribution theory, and minimal surface theory. Furthermore, we provide a systematic description of the relationship among the Montel theorem, the Liouville theorem and the Bernstein theorem, as well as the Carathéodory–Montel theorem, the Picard little theorem and the Fujimoto theorem. We refer to this theoretical framework as the Bloch–Ros principle. We further generalize this principle to various classes of surfaces, including maxfaces in the Lorentz–Minkowski 3-space and improper affine fronts in the affine 3-space. Furthermore, we present recent progress on the application of the Bloch–Ros principle to the ramification theorem, providing a unified perspective on phenomena concerning totally ramified values. This presentation is based on joint work with Yu Kawakami and recent independent research.

Riku Kishida (Institute of Science Tokyo)

Zero mean curvature surfaces with singularities in the isotropic 3-space

Abstract

The isotropic 3-space I^3 is the 3-dimensional real vector space with the degenerate inner product of signature $(0++)$. For a space-like surface in I^3, the mean curvature function can be defined in a natural way. A space-like surface whose mean curvature function vanishes everywhere is called a zero mean curvature surface. A zero mean curvature surface in I^3 admits a Weierstrass-type representation formula. In this presentation, we introduce a class of zero mean curvature surfaces with singularities in I^3, which we call ZMC-faces. As an application of ZMC-faces, we show that several Osserman-type inequalities can be obtained under certain assumptions on both completeness and finiteness of the total curvature. We also present some examples which attain the equality in these inequalities.

Hikozo Kobayashi (Hiroshima University)

Constant curvature and doubly totally geodesic hypersurfaces in homogeneous statistical manifolds

Abstract

A statistical manifold is a certain generalization of a Riemannian manifold. In statistical geometry, several analogues of constant curvature and totally geodesic submanifolds have been introduced and studied. On the other hand, for a certain class of homogeneous Riemannian manifolds, including simply connected irreducible Riemannian symmetric spaces, it is known that having constant curvature is equivalent to admitting a totally geodesic hypersurface. Moreover, such a hypersurface is essentially unique.

In this poster presentation, we investigate whether an analogous phenomenon occurs in the geometry of homogeneous statistical manifolds. In particular, we consider left-invariant statistical structures on the solvable Lie group $G_{\mathbb{R}\mathrm{H}^n}$, equipped with a left-invariant Riemannian metric for which it is isometric to the real hyperbolic space $\mathbb{R}\mathrm{H}^n$. We show that, on this Lie group, constant curvature in the sense of Kurose implies the existence of a codimension-one Lie subgroup that is doubly totally geodesic in the sense of Furuhata [Inf. Geom. (2024)] and is essentially unique. The converse, however, fails. There are infinitely many inequivalent structures that admit such a Lie subgroup but do not have constant curvature. Moreover, if doubly total geodesicity is weakened to total geodesicity with respect to the statistical connection, the uniqueness of totally geodesic hypersurfaces is lost.

This poster presentation is based on joint work with Akira Kubo (Hiroshima Institute of Technology) and Hiroshi Tamaru (Osaka Metropolitan University).

Motoki Masada (Kyushu University)

Michell–Prager type truss structures constructed from integrable discrete power function and discrete logarithmic function

Abstract

The Michell–Prager type truss structures are constructed from the integrable discrete power and logarithmic functions. It is demonstrated that specific sublattices, subject to suitable boundary conditions, yield approximate Michell trusses, which are theoretically optimal structures achieving force equilibrium with material economy. The result of shape optimization minimizing the Michell functional is provided for numerical evidence of their optimality.

Fumika Mizoguchi (RIKEN Center for Advanced Intelligence Project)

Geometric structures on nilpotent and solvable Lie algebras obtained by quivers

Abstract

We are interested in when a given Lie group admits a special left-invariant geometric structure. In particular, an important problem is to study the relationship between the existence of such structures on Lie groups and the algebraic properties of the underlying Lie algebras. In this talk, we construct nilpotent and solvable Lie algebras from finite quivers without cycles. Furthermore, we prove that the simply-connected nilpotent and solvable Lie groups corresponding to the nilpotent and solvable Lie algebras obtained by quivers admit Ricci solitons. Moreover, we show that if a quiver is an oriented multi-tree, the obtained solvable Lie group admits a metric that is isometric to the direct product of a flat metric and an Einstein metric.

Hidehito Nagao (Gifu Shotoku Gakuen University)

Rational Surface Realizations and Birational Transformations of q-Painlevé Equations with Surface Type $A_1^{(1)}$ and Symmetry Type $E_7^{(1)}$

Abstract

We study three rational surface realizations of the q-Painlevé equation with surface type $A_1^{(1)}$ and symmetry type $E_7^{(1)}$, obtained as degeneration limits of the q-Painlevé equation with surface type $A_0^{(1)}$ and symmetry type $E_8^{(1)}$. Although these realizations have the same surface type $A_1^{(1)}$, they possess different configurations of accessible singular points and induce different birational representations of the affine Weyl group $E_7^{(1)}$. We also describe explicit birational transformations relating these realizations and clarify the geometric relationships among the corresponding dynamical systems.

Shuki Sano (Institute of Science Tokyo)

Approximation and Interpolation Theorem for Maximal Surfaces with Singularities

Abstract

Alarcón, Forstnerič, and López established approximation and interpolation theorems for conformal minimal surfaces using the Enneper-Weierstrass representation formula. In this presentation, we present an approximation and interpolation theorem for maxfaces, which is obtained by applying their methods to maxfaces (maximal surfaces with singularities in the Lorentz-Minkowski 3-space) defined by Umehara and Yamada. Furthermore, we explain that by incorporating the singularity criteria for maxfaces into the arguments, assertions concerning singularities can be obtained in addition to approximation and interpolation. Moreover, as corollaries of this theorem, we discuss the existence of a maxface with prescribed singularities at specified points, as well as the existence of a maxface whose singular set has a dense image in the Lorentz-Minkowski 3-space.

Mario Julián Rodríguez Sánchez de Toca (University of Santiago de Compostela)

Curvature-adapted homogeneous hypersurfaces in symmetric spaces of non-compact type

Abstract

In the context of Riemannian manifolds, there are two operators that give the intrinsic and extrinsic curvature of a submanifold: the Jacobi operator and the shape operator, respectively. We say that a hypersurface is curvature-adapted to its ambient Riemannian manifold if there is a basis where these two operators diagonalize simultaneously, or equivalently, they commute.

In this poster we will analyze the problem of classifying curvature-adapted hypersurfaces in the context of irreducible symmetric spaces of non-compact type with the extra hypothesis of being homogeneous, using cohomogeneity one actions as our main tool.

Shogo Shimada (Tokushima University)

Bäcklund transformations for curves and their applications to semi-discrete K-surfaces

Abstract

Transformation theory of surfaces plays an important role in the study of integrable systems. We first focus on Bäcklund transformations for surfaces with constant negative Gaussian curvature (K-surfaces). Bäcklund transformations provide new K-surfaces from a given one and, at the same time, generate new solutions to the sine-Gordon equation – the structure equation of K-surfaces – from known ones.

We then introduce Bäcklund transformations for curves with constant nonzero torsion. We show that these transformations give rise to two types of semi-discrete integrable equations, generalizing results obtained by Calini and Ivey. Furthermore, we derive superposition principles

for solutions of these equations through the geometry of semi-discrete K-surfaces. Finally, we present examples of semi-discrete K-surfaces constructed via these superposition principles. This is joint work with Masashi Yasumoto.

Naoya Suda (Kobe University)

Transformations and discretization of spacelike K-surfaces

Abstract

We extend the transformation theory for surfaces with constant negative Gaussian curvature in Euclidean space, classically studied by Bäcklund and Bianchi, to surfaces with constant negative extrinsic Gaussian curvature in Minkowski space by referring to Gu, Hu, and Zhou (2005). This poster presents results for cases that result in Ribaucour transformations as well as examples of the transformed surfaces. Moreover, we introduce a discretization based on this transformation.

Homare Tadano (Yamaguchi University)

A zoo of Myers-type theorems

Abstract

The classical Myers theorem has inspired many refinements and generalizations. In this poster, we present a brief tour of the zoo of Myers-type theorems. Like animals in a zoo, Myers-type theorems come in many species, each characterized by its own curvature assumptions, yet all leading to the compactness of manifolds. Some species are already well domesticated, while others are rarely seen in the wild.

We begin with classical Myers-type theorems and then discuss several recent refinements and generalizations obtained by the presenter. Admission is free.

Keita Takahashi (Institute of Science Tokyo)

Completeness conditions for globally hyperbolic spacetimes

Abstract

In this talk, we discuss several completeness conditions for globally hyperbolic spacetimes within the framework of low-regularity settings. These conditions, originally introduced by Busemann and Beem, serve as Lorentzian analogues to those found in the classical Hopf-Rinow theorem. As a related result, we explain an approximation theorem for the space of Cauchy hypersurfaces, a space whose properties have recently been investigated by Lange and Peteranderl.

Sakuma Takeshita (Tokushima University)

Iterated minimal Darboux transformations

Abstract

Establishing a permutability theorem of transformations for surfaces is crucially important in constructing new surfaces and studying integrable systems. We first focus on isothermic surfaces. Isothermic surfaces include constant mean curvature surfaces and minimal surfaces, and are known to admit various transformations. These transformations not only characterize isothermic surfaces, but also enable us to obtain new isothermic surfaces from known ones.

In this poster, using a quaternionic calculus, we introduce the permutability of Christoffel, Goursat, and Darboux transformations for isothermic surfaces in the Euclidean space. We show that a superposition principle of the Darboux transformation for minimal surfaces can be obtained by extending to higher-dimensional permutability of transformations for isothermic surfaces. Furthermore, we illustrate concrete examples of applying the Enneper Surface and the catenoid, as well as the results obtained regarding their asymptotic behavior. This is joint work with Masashi Yasumoto.

Yuta Yamauchi (Yokohama National University)

Minimal total absolute curvature for equiaffine immersions

Abstract

For immersions of compact n-dimensional manifolds into Euclidean spaces, the total absolute curvature is a global geometric quantity defined by integrating the absolute value of the Lipschitz–Killing curvature. The Chern–Lashof theorem states that the total absolute curvature is bounded below by the sum of the Betti numbers. Moreover, it is equal to 2 if and only if the image is a convex hypersurface embedded in an $(n+1)$-dimensional affine subspace. In 2001, Koike introduced the total absolute curvature for equiaffine immersions of arbitrary dimension and codimension, and established a Chern–Lashof type inequality. However, a geometric characterization of the case in which the total absolute curvature attains its minimum value had remained unknown in the equiaffine setting. In this poster, I investigate the relationship between the minimality of the total absolute curvature and convexity for equiaffine immersions, without assuming the non-degeneracy of the affine fundamental form. I prove that, as in the Euclidean case, an equiaffine immersion has total absolute curvature 2 if and only if its image is a convex hypersurface embedded in an $(n+1)$-dimensional affine subspace.

Yuri Yamashita (Osaka Metropolitan University)

Geometry of left-invariant pseudo-Riemannian metrics on Lie groups and closed orbit spaces

Abstract

In geometry, the classification of distinguished metrics is an important issue. In this research, we consider distinguished left-invariant pseudo-Riemannian metrics on Lie groups. It is a natural and important problem to determine whether a given Lie group G admits some distinguished left-invariant pseudo-Riemannian metrics or not. For a given Lie group, there is a natural group action on the space of left-invariant pseudo-Riemannian metrics by the scalar multiplication and automorphisms. The orbit space of this group action is called the moduli space. However, in general, the moduli space is not Hausdorff and is complex. Therefore, in this research, we particularly focus on the closed orbit space among these orbit spaces. In this study, we introduce that it is sufficient to consider metrics only on the closed orbit space to study existence and nonexistence of distinguished metrics. Additionally, we have determined the closed orbit space for some Lie groups.