J. Math. Tokushima Univ.
Vol. 49 (2015), 13–22

Existence and Uniqueness results of Some Nonlinear Parabolic Equations with Uniformly Continuous Data

BY

Masaki Ohnuma

Department of Mathematical Sciences,
Faculty of Integrated Arts and Sciences,
Tokushima University, Tokushima 770-8502, JAPAN
e-mail address : *ohnuma@tokushima-u.ac.jp*
(Received October 16, 2015)

Abstract

In this note, we consider the Cauchy problem of nonlinear degenerate parabolic equations including the level set equation of the mean curvatue equation and the p-Laplace diffusion equation with $p \geq 2$. We shall give existence and uniqueness results to such equations provided that the initial data is uniformly continuous.

2010 Mathematics Subject Classification. 35K65, 35D40, 35A01, 35A02, 35K92, 35K93.

Introduction

We consider the Cauchy problem of nonlinear degenerate parabolic equations of the form

$$u_t + F(\nabla u, \nabla^2 u) = 0 \quad \text{in} \quad Q_T := (0,T) \times \mathbf{R}^N, \tag{1}$$

$$u(0,x) = a(x) \quad \text{on} \quad \mathbf{R}^N, \tag{2}$$

where $u : \overline{Q_T} \to \mathbf{R}$ is an unknown function, $F = F(q,X)$ is a given function, $a(x)$ is uniformly continuous and $T > 0$. Here $u_t = \partial u/\partial t$, ∇u and $\nabla^2 u$ denote, respectively, the time derivative of u, the gradient of u and the Hessian of u in space variables. The function $F = F(q,X)$ needs not to be *geometric* in the sense of Chen, Giga and Goto [1], i.e.,

$$F(\lambda q, \lambda X + \mu q \otimes q) = \lambda F(q,X) \quad \text{for all} \quad \lambda > 0, \mu \in \mathbf{R}, q \in \mathbf{R}^N \backslash \{0\}, X \in \mathbf{S}^N,$$

where $\mathbf{S}^N$ denotes the space of all real symmetric matrices with order N.

A typical example of (1) we consider is the p-Laplace diffusion equation

$$u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0 \quad \text{in} \quad Q_T, \tag{3}$$

with $p \geq 2$. For this equation $F = F(q, X)$ is given by

$$F(q, X) = -|q|^{p-2}\text{trace}\left\{\left(I + (p-2)\frac{q \otimes q}{|q|^2}\right)X\right\}, \tag{4}$$

where $\otimes$ denotes the tensor product.

A comparison principle for (1) was established by the author and K. Sato [9]. Once the comparison principle for (1) was proved, we can construct the unique global-in-time viscosity solution of (1)-(2) with bounded uniformly continuous data (cf. [9]).

One can improve the proof of the unique existence theorem of (1)-(2) when the initial data is uniformly continuous on $\mathbf{R}^N$. For the proof we take similar procedures as in [9]. We have to modify the proof [9, Lemma 4.5, 4.6] since the initial data is not bounded. Moreover, we have to prepare a comparison principle for (1) to unbounded solutions. When we can improve the lemmas and a comparison principle, we conclude the same unique existence theorem of (1)-(2) for any uniformly continuous initial data $a(x)$.

Here we shall write a little bit generalized equation of (3)

$$u_t - |\nabla u|^{p-2}\text{trace}\left\{\left(I + (p'-2)\frac{\nabla u \otimes \nabla u}{|\nabla u|^2}\right)\nabla^2 u\right\} = 0 \quad \text{in} \quad Q_T, \tag{5}$$

where $p' \geq 1$ and $p \geq 2$. For this equation

$$F(q, X) = -|q|^{p-2}\text{trace}\left\{\left(I + (p'-2)\frac{q \otimes q}{|q|^2}\right)X\right\}. \tag{6}$$

The equation (5) has interesting examples.

Example 1. If $p = p'$ then (5) is nothing but the p-Laplace diffusion equation (3)

$$u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0 \quad \text{in} \quad Q_T.$$

The p-Laplace diffusion equation has been studied by many authors. Although we use viscosity solutions as a weak solution (cf. [2, 5]), many authors are familiar with usual weak solutions defined in distribution sense, since the p-Laplace diffusion equation has the divergence structure. In fact, our unique existence theorem has already been known by interpreting solutions as usual weak solutions. However, the proof of the continuity of such a weak solution needs many procedures, since it was done by using the Harnak inequality and many a priori estimates. For details, we refer to the book by DiBenedetto [3]. Our procedures are based on Perron's method, so the proof is simpler than that of usual one.

Note that the equation (3) is not geometric.
Example 2. If $p = 2$ and $p' = 1$ then (5) is the level set mean curvature flow equation

$$u_t - |\nabla u| \mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right) = 0 \quad \text{in} \quad Q_T. \tag{7}$$

This equation was initially studied by Chen, Giga and Goto [1] and Evans and Spruck [4]. They established the comparison principle and proved the unique existence theorem of (7)-(2), independently. In [1] they consider more general equations (1). To establish the comparison principle they assume $F = F(q,X)$ can be extended continuously at $(q,X) = (0,O)$, i.e., $-\infty < F_*(0,O) = F^*(0,O) < +\infty$, especially F of (7) satisfies $F_*(0,O) = F^*(0,O) = 0$. Here $F_*(q,X)$ and $F^*(q,X)$ denotes the upper and lower semicontinuous envelope of $F(q,X)$, respectively (cf. [1]). The equation (7) does not have the divergence structure. So the theory of usual weak solution does not apply to (7). This situation is different from that of (3) and (7) is geometric.

1 Definition of viscosity solutions and a comparison theorem

Here and hereafter we shall study a general equation of form

$$u_t + F(\nabla u, \nabla^2 u) = 0 \quad \text{in} \quad Q_T. \tag{1.1}$$

We list assumptions on $F = F(q,X)$.
(F1) F is continuous in $(\mathbf{R}^N \backslash \{0\}) \times \mathbf{S}^N$.
(F2) F is degenerate elliptic, i.e.,

$$\text{if} \quad X \geq Y \quad \text{then} \quad F(q,X) \leq F(q,Y) \quad \text{for all} \quad q \in \mathbf{R}^N \backslash \{0\}.$$

(F3) $F_*(0,O) = F^*(0,O) = 0$.
(F4) For every $R > 0$,

$$c_R = \sup\{|F(q,X)|; |q| \leq R, |X| \leq R, q \neq 0\} < +\infty.$$

Remark 1.1. For the levet set mean curvature flow equation,

$$|F(q,X)| \leq R(N+1).$$

This $F(q,X)$ satisfies (F4). For (6) with $p' \geq 1$ and $p \geq 2$,

$$|F(q,X)| \leq |q|^{p-2} RN + |p'-2||q|^{p-2} R.$$

When $p \geq 2$, we have

$$|F(q,X)| \leq R^{p-1}(N + |p'-2|).$$

This $F(q,X)$ satisfies (F4). To define viscosity solutions we have to prepare a class of "test functions". This class is important and a part of test functions as space variable functions.

Definition 1.2. We denote by $\mathcal{F}(F)$ the set of function $f \in C^2[0,\infty)$ which satisfies

$$f(0) = f'(0) = f''(0) = 0, \quad f''(r) > 0 \quad \text{for all} \quad r > 0 \tag{1.2}$$

and

$$\lim_{|x|\to 0, x\neq 0} F\big(\pm\nabla f(|x|), \pm\nabla^2 f(|x|)\big) = 0. \tag{1.3}$$

Remark 1.3. Our definition of $\mathcal{F}(F)$ is an extension of that in [7]. Actually, if F is geometric then the set $\mathcal{F}(F)$ is the same in [7].

For F of (6) with $p' \geq 1$, we shall write an example $f \in \mathcal{F}(F)$ if it is possible.

(i) If $1 < p < 2$ then $f(r) = r^{1+\sigma}$ with $\sigma > 1/(p-1) > 1$.
(ii) If $p \geq 2$ then $f(r) = r^4$.
(iii) If $p \leq 1$ then $\mathcal{F}(F)$ is empty.

On the other hand, if F is geometric then $\mathcal{F}(F)$ is not empty (cf. [7]). We shall define a class of test function so called admissible.

Definition 1.4. A function $\varphi \in C^2(Q_T)$ is *admissible* (in short $\varphi \in \mathcal{A}(F)$) if for any $\hat{z} = (\hat{t}, \hat{x}) \in Q_T$ with $\nabla\varphi(\hat{z}) = 0$, there exist a constant $\delta > 0$, $f \in \mathcal{F}(F)$ and $\omega \in C[0,\infty)$ satisfying $\omega \geq 0$ and $\lim_{r\to 0}\omega(r)/r = 0$ such that

$$|\varphi(z) - \varphi(\hat{z}) - \varphi_t(\hat{z})(t-\hat{t})| \leq f(|x-\hat{x}|) + \omega(|t-\hat{t}|) \tag{1.4}$$

for all $z = (t,x)$ with $|z - \hat{z}| < \delta$. Now we shall introduce a notion of viscosity solutions of (1.1).

Definition 1.5. Assume that (F1) and (F2) hold and that $\mathcal{F}(F)$ is not empty.

1. A function $u : \overline{Q_T} \to \mathbf{R} \cup \{-\infty\}$ is a viscosity subsolution of (1.1) if u^* is locally bounded from above in $\overline{Q_T}$ and for all $\varphi \in \mathcal{A}(F)$ and all local maximum point z of $u^* - \varphi$ in Q_T,

$$\begin{cases} \varphi_t(z) + F(\nabla\varphi(z), \nabla^2\varphi(z)) & \leq 0 \qquad \text{if} \quad \nabla\varphi(z) \neq 0, \\ \varphi_t(z) & \leq 0 \qquad \text{otherwise.} \end{cases}$$

2. A function $u : \overline{Q_T} \to \mathbf{R} \cup \{+\infty\}$ is a viscosity supersolution of (1.1) if u_* is locally bounded from below in $\overline{Q_T}$ and for all $\varphi \in \mathcal{A}(F)$ and all local minimum point z of $u_* - \varphi$ in Q_T,

$$\begin{cases} \varphi_t(z) + F(\nabla\varphi(z), \nabla^2\varphi(z)) & \geq 0 \qquad \text{if} \quad \nabla\varphi(z) \neq 0, \\ \varphi_t(z) & \geq 0 \qquad \text{otherwise.} \end{cases}$$

3. A function u is called a viscosity solution of (1.1) if u is both a viscosity sub- and super-solution of (1.1). We often suppress the word "viscosity" except

in statements of theorems. Before we shall explain a comparison theorem, we need an additional assumption on F.
(F5) (i) $\mathcal{F}(F)$ is not empty. (ii) If $f \in \mathcal{F}(F)$ then $af \in \mathcal{F}(F)$ for all $a > 0$.
Remark 1.6. (i) When $p > 1$ and $p' \geq 1$, F of (6) satisfies (F1), (F2) and (F5).
(ii) If F is geomtric, then (F1), (F2) and (F5) hold. Here we introduce a nice comparison principle by Giga, Goto, Ishii and Sato [6]. Their comparison principle use usual viscosity solutions (cf. [2]). By the aid of Giga's book [5], under our assumptions on F we know our viscosity subsolutions and supersolutions are usual viscosity subsolutions and supersolutions, respectively. We can apply the comparison principle in [6].

Theorem 1.7. *(Comparison theorem) [6, Theorem 2.1]. Suppose that F satisfies (F1), (F2), (F3) and (F4). Let u and v be upper semicontinuous and lower semicontinuous on $[0, T) \times \mathbf{R}^N$, respectively. Let u and v be a viscosity sub- and super-solution of (1.1), respectively. Assume that*

(A1) $u(t,x) \leq K(|x|+1)$, $v(t,x) \geq -K(|x|+1)$ for some $K > 0$ independent of $(t,x) \in Q_T$;

(A2) there is a modulus m such that

$$u(0,x) - v(0,y) \leq m(|x-y|) \quad \text{for all} \quad (x,y) \in \mathbf{R}^N \times \mathbf{R}^N;$$

(A3) $u(0,x) - v(0,y) \leq K(|x-y|+1)$ on $\mathbf{R}^N \times \mathbf{R}^N$ for some $K > 0$ independent of (x,y). Then there is a modulus m such that

$$u(t,x) - v(t,y) \leq m(|x-y|) \quad \text{on} \quad (0,T) \times \mathbf{R}^N \times \mathbf{R}^N.$$

In particular $u(t,x) \leq v(t,x)$ on Q_T.

2 Construction of solutions

We shall construct a viscosity solution to the Cauchy problem of (1)-(2). Our construction of solutions is based on Perron's method. The pocedure is the same as in [9] so we omit the proofs. For details see [9].

As usual we obtain the following two propositions. We state them without the proof.

Proposition 2.1 (9, Proposition 2.5)**.** *Assume that (F1), (F2) and (F5) hold. Let $\mathcal{S}$ be a set of subsolutions of (1). We set*

$$u(z) := \sup\{v(z); v \in \mathcal{S}\}, \quad z \in Q_T.$$

If u^* is locally bounded from above in $\overline{Q_T}$, then u is a subsolution of (1).

A similar assertion holds for supersolutions of (1).

Proposition 2.2 (9, Proposition 2.6)**.** *Assume that (F1), (F2) and (F5) hold. Let $\mathcal{S}$ be a set of subsolutions of (1). Let ℓ and h be a subsolution and a supersolution of (1), respectively. Assume that ℓ and h are locally bounded in $\overline{Q_T}$ and $\ell \le h$ holds. We set*

$$u(z) := \sup\{v(z); v \in \mathcal{S}, \ell \le v \le h \quad in \quad Q_T\}, \quad z \in Q_T.$$

Then u is a solution of (1).

To construct a solution we only have to find a sub- and a super-solution, respectively, which fulfills the hypotheses of Proposition 2.2 and the given initial data $a(x)$. From the degenerate elliptic condition (F2), we have a sufficient condition that a C^2 function to be a super- and a sub-solution, respectively.

Lemma 2.3. *Assume that F satisfies (F1), (F2). Suppose that $\mathcal{F}(F)$ is not empty. If $u \in C^2(Q_T)$ satisfies*

$$\begin{cases} u_t(z) + F(\nabla u(z), \nabla^2 u(z)) & \ge 0 \qquad \textit{if} \quad \nabla u \ne 0, \\ u_t(z) & \ge 0 \qquad \textit{otherwise}, \end{cases}$$

$$\left(\textit{resp.} \quad \begin{cases} u_t(z) + F(\nabla u(z), \nabla^2 u(z)) & \le 0 \qquad \textit{if} \quad \nabla u \ne 0, \\ u_t(z) & \le 0 \qquad \textit{otherwise}, \end{cases} \right)$$

then u is a viscosity supersolution (resp. subsolution) of (1).

Here we shall write down an outline of construction of a solution of (1)-(2).
(a) Introduction of $\mathcal{G}$ (a family of C^2 functions).
(b) Construction of C^2 typical subsolutions and supersolutions of (1), respectively. These are of form: (function of the time variable)+(function of the space variable) and (function of the space variable)$\in \mathcal{G}$.
(c) Construction of a subsolution and a supersolution of (1)-(2), respectively. Here we will use Proposition 2.1.
(d) We shall check the hypotheses of Proposition 2.2.
(e) Finally, we can construct a solution of (1)-(2) by using Propositon 2.2.

Now we shall carry out all steps.
(a) We introduce a set of C^2 functions $\mathcal{G}$;

$$\mathcal{G} := \{g \in C^2[0,\infty); g(0) = g'(0) = 0, g'(r) > 0 \ (r > 0), \lim_{r \to 0} g(r) = +\infty\}.$$

Remark 2.4. (i) If $g(r) \in \mathcal{G}$ then $g(|x|) \in C^2(\mathbf{R}^N)$. A direct calculation yields

$$\nabla^2 g(|x|) = \frac{g'(|x|)}{|x|} I + \left(g''(|x|) - \frac{g'(|x|)}{|x|} \right) \left(\frac{x}{|x|} \otimes \frac{x}{|x|} \right).$$

Although $\nabla^2 g(|x|)$ does not appear to be continuous at $x = 0$, it is regarded as a continuous function. Indeed, $\nabla^2 g(0) = g''(0) I$ holds since $\lim_{r \to 0} g'(r)/r = g''(0)$ by the definition of $\mathcal{G}$.

(ii) If $f(r) \in \mathcal{F}(F)$ then $f(r) \in \mathcal{G}$.
(iii) We may assume that

$$\sup_{r\geq 0} g'(r) < +\infty, \quad \sup_{r\geq 0} g''(r) < +\infty.$$

(b) We observe nice properties of F, which is important to construct a sub- and a super-solution, respectively.

Lemma 2.5 (9, Lemma 4.3)**.** *Assume that F satisfies (F1), (F2) and (F5). Then the following properties hold.*
(F6)$_+$ There exists $g \in \mathcal{G}$ such that for each $A > 0$, there exists $B > 0$ that satisfies

$$F\big(\nabla(Ag(|x|)), \nabla^2(Ag(|x|))\big) \geq -B \quad \textit{for all} \quad x \in \mathbf{R}^N\backslash\{0\}. \tag{2.1}$$

(F6)$_-$ There exists $g \in \mathcal{G}$ such that for each $A > 0$, there exists $B > 0$ that satisfies

$$F\big(\nabla(-Ag(|x|)), \nabla^2(-Ag(|x|))\big) \leq B \quad \textit{for all} \quad x \in \mathbf{R}^N\backslash\{0\}. \tag{2.2}$$

Then we obtain the following by Lemma 2.3.

Lemma 2.6 (9, Lemma 4.4)**.** *Assume that F satisfies (F1), (F2) and (F5). Then $u_+(t,x) := Bt + Ag(|x|)$ and $u_-(t,x) := -Bt - Ag(|x|)$ is a viscosity supersolution and a subsolution of (1), respectively, where g, A and B are appeared in (F6)$_+$ and (F6)$_-$.*

(c) Since the equation (1) is invariant under the translation and addition of constants, we know $u_{+,\xi}(t,x;\varepsilon) := a(\xi)+Bt+Ag(|x-\xi|)+\varepsilon$ is a supersolution of (1) and $u_{-,\xi}(t,x;\varepsilon) := a(\xi) - Bt - Ag(|x-\xi|) - \varepsilon$ is a subsolution of (1) for each $\varepsilon > 0$ and $\xi \in \mathbf{R}^N$, where g, A, B are appeared in (F6)$_+$ and (F6)$_-$, respectively.

Up to now we only consider the equation (1). We shall construct a supersolution and a subsolution of (1)-(2), respectively. We shall explain how to construct a supersolution of (1) satisfying the initial data. This is only new parts compared with [9] because $a(x)$ is not bounded. We can construct a subsolution by similar procedure.

Lemma 2.7 (8, Lemma 3.7)**.** *Suppose that $a(x)$ is a given uniformly continuous function on $\mathbf{R}^N$ (in short $a(x) \in UC(\mathbf{R}^N)$). For all $\varepsilon > 0$ with $0 < \varepsilon < 1$, there exist $A(\varepsilon) > 0$ and $B(\varepsilon) > 0$ such that for each $\xi \in \mathbf{R}^N$*

$$u_{+,\xi}(0,x;\varepsilon) \geq a(x) \quad \textit{for all} \quad x \in \mathbf{R}^N \tag{2.3}$$

and

$$\inf_{\xi\in\mathbf{R}^N} u_{+,\xi}(0,x;\varepsilon) \leq a(x) + \varepsilon \quad \textit{for all} \quad x \in \mathbf{R}^N. \tag{2.4}$$

Proof. It is easy to show (2.4). We put $\xi = x$ in the left side of (2.4) and observe that

$$\inf_{\xi \in \mathbf{R}^N} u_{+,\xi}(0, x; \varepsilon) \le a(x) + \varepsilon.$$

To prove the inequality (2.3) we have to show the existence of $A(\varepsilon)$ such that

$$|a(x) - a(\xi)| \le A(\varepsilon) g(|x - \xi|) + \varepsilon. \tag{2.5}$$

Since $a(x) \in UC(\mathbf{R}^N)$, there exist a concave modulus function m (i.e., $m[0,\infty) \to [0,\infty)$ is continuous, nondecreasing and $m(0) = 0$) such that

$$|a(x) - a(y)| \le m(|x - y|) \quad \text{for all} \quad x, y \in \mathbf{R}^N.$$

Since m is concave, for each $\varepsilon > 0$ there exists a constant $M(\varepsilon) > 0$ such that

$$m(r) \le M(\varepsilon) r + \varepsilon/2 \quad \text{for all} \quad r \in [0, \infty).$$

Then we take $A(\varepsilon)$ so that

$$M(\varepsilon) r + \varepsilon/2 \le A(\varepsilon) g(r) + \varepsilon \quad \text{for all} \quad r \in [0, \infty).$$

Thus we obtain (2.5) which yields the inequality (2.3). □

We can prove the following by a similar argument.

Lemma 2.8 (8, Lemma 3.8)**.** *Suppose that $a(x)$ is a given uniformly continuous function on $\mathbf{R}^N$ (in short $a(x) \in UC(\mathbf{R}^N)$). For all $\varepsilon > 0$ with $0 < \varepsilon < 1$, there exist $A(\varepsilon) > 0$ and $B(\varepsilon) > 0$ such that for each $\xi \in \mathbf{R}^N$*

$$u_{-,\xi}(0, x; \varepsilon) \le a(x) \quad \textit{for all} \quad x \in \mathbf{R}^N \tag{2.6}$$

and

$$\sup_{\xi \in \mathbf{R}^N} u_{-,\xi}(0, x; \varepsilon) \ge a(x) - \varepsilon \quad \textit{for all} \quad x \in \mathbf{R}^N. \tag{2.7}$$

Now by Proposition 2.1 we conclude

Lemma 2.9 (9, Lemma 4.7)**.** *Assume that F satisfies (F1), (F2) and (F5). Suppose that $a(x) \in UC(\mathbf{R}^N)$. Then for all $T > 0$, there exist $U_+, U_- : [0, T] \times \mathbf{R}^n \to \mathbf{R}$ such that U_+ is a supersolution of (1)-(2), U_- is a subsolution of (1)-(2) and $(U_+)_*(0, x) = (U_-)^*(0, x) = a(x)$. Moreover, $U_+(t, x) \ge U_-(t, x)$ in $\overline{Q_T}$.*

Sketch of proof. By Proposition 2.1

$$U_+(t, x) := \inf\{u_{+,\xi}(t, x; \varepsilon); 0 < \varepsilon < 1, \xi \in \mathbf{R}^N\} \tag{2.8}$$

is also a supersolution of (1). Applying Lemma 2.7 we observe that $U_+(0,x) = a(x)$ for all $x \in \mathbf{R}^N$. Moreover, since $a(x) \le (U_+)_*(0,x) \le U_+(0,x) = a(x)$, we see $(U_+)_*(0,x) = a(x)$. For a subsolution we set

$$U_-(t,x) := \sup\{u_{-,\xi}(t,x;\varepsilon); 0 < \varepsilon < 1, \xi \in \mathbf{R}^N\}. \tag{2.9}$$

By the definition of U_+ and U_-, we see $U_+(t,x) \ge U_+(0,x) = a(x) = U_-(0,x) \ge U_-(t,x)$ in $\overline{Q_T}$. □

Thus we constructed a supersolution and a subsolution of (1)-(2), respectively.

(d) To construct a solution of (1)-(2) we have to check that the supersolution U_+ and the subsolution U_-, respectively, fulfills the hypotheses of Proposition 2.2.

Lemma 2.10 (cf. 9, Lemma 4.8)**.** *Assume that F satisfies (F1), (F2) and (F5). Suppose that $a(x) \in UC(\mathbf{R}^N)$. Let U_+ and U_- be as in Lemma 2.9. Then there is a modulus function such that*

$$U_+(t,x) - U_-(0,y) \le \omega(|x-y| + t) \quad \textit{for all} \quad t \in [0,T], x, y \in \mathbf{R}^N \tag{2.10}$$

and

$$U_+(0,x) - U_-(s,y) \le \omega(|x-y| + s) \quad \textit{for all} \quad s \in [0,T], x, y \in \mathbf{R}^N. \tag{2.11}$$

Moreover, U_+ is locally bounded from above and U_- is locally bounded from below in $\overline{Q_T}$.

Note that the inequality (2.10) and (2.11) imply that U_+ and U_- fulfills (A1), (A2) and (A3).

(e) Finally, by Proposition 2.2 we can construct a solution of (1)-(2).

The uniqueness of solutions of (1)-(2) comes from the Comparison theorem. So we only have to check conditions (A1)-(A3) to U_+ and U_- in Lemma 2.9.

Now, we conclude

Theorem 2.11. *Suppose that F satisfies (F1), (F2), (F3), (F4) and (F5). Assume that $a(x) \in UC(\mathbf{R}^N)$. Then there exists a (unique) viscosity solution $u \in UC([0,T) \times \mathbf{R}^N)$ of (1)-(2).*

In particular, we obtain a corollary;

Corollary 2.12. *Assume that $a(x) \in UC(\mathbf{R}^N)$. Then there exists a (unique) viscosity solution $u \in UC([0,T) \times \mathbf{R}^N)$ of (5)-(2) with $p' \ge 1$ and $p \ge 2$.*

References

[1] Y.-G. Chen, Y. Giga and S. Goto, *Uniqueness and existence of viscosity solutions of generalized mean curvature flow equations*, JḊiffĠeometry, **33**, (1991), 749–786.

[2] M. G. Crandall, H. Ishii and P. L. Lions, *User's guide to viscosity solutions of second order partial differential equations*, BullȦmerṀathṠoc., **27**, (1992), 1–67.

[3] E. DiBenedetto, Degenerate parabolic equations, Springer–Verlag, 1993.

[4] L. C. Evans and J. Spruck, *Motion of level sets by mean curvature I*, J. DiffĠeometry, **33**, (1991), 635–681.

[5] Y. Giga, Surface Evolution Equations, Birkhäuser, 2006.

[6] Y. Giga, S. Goto, H. Ishii and M.-H. Sato, *Comparison principle and convexity preserving properties for singular degenerate parabolic equations on unbounded domains*, Indiana Univ. Math. J., **40**, (1991), 443–470

[7] H. Ishii and P. E. Souganidis, *Generalized motion of noncompact hypersurfaces with velocity having arbitrary growth on the curvature tensor*, Tôhoku MathJ̇., **47**, (1995), 227–250.

[8] M. Ohnuma, *Some remarks on singular degenerate parabolic equations including the p-Laplace diffusion equation*, Pitman Research Notes in Mathematics Series, **384**, (1998), 54–65.

[9] M. Ohnuma and K. Sato, *Singular degenerate parabolic equations with applications to the p-Laplace diffusion equation*, Comm. P. D. E., **22**, (1997), 381–411.