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Erratum to "Blowup Phenomena for Nonlinear Dissipative Wave Equations", J. Math. Tokushima Univ., 30(1996), 19–34

By

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p.25 Theorem 2 should be changed into the following :

Theorem 2. ($\delta_2 > 0, \delta_3 = 0$) *Let $\delta_2 > 0$ and $\delta_3 = 0$ in (0.1). Suppose that $\alpha > \beta$ and*

$$E(0) < 0 \quad and \quad G(0) \equiv (-E(0))^{\omega} + 2\omega m_0^{-1}(u_0, u_1) > 0 .$$

Then there exists a T such that

$$0 < T \le m_0 m_1 \omega (1-\omega)^{-1} G(0)^{-(1-\omega)/\omega}$$

and the local solution $u(t)$ in the sense of Theorem 1 blows up at the finite time T, where ω, m_0, and m_1 are positive constants such that

$$\omega = 1 - (\frac{1}{\beta+2} - \frac{1}{\alpha+2}) \quad (1/2 < \omega < 1),$$
$$m_0 = (2\delta_2(1+2/\alpha)|\Omega|^{\frac{\alpha-\beta}{\alpha+2}}(-E(0))^{-(1-\omega)})^{1/(\beta+1)},$$
$$m_1 = 2\max\Big\{1, (4(\alpha+2)/\alpha)\{(2|\Omega|^{\frac{\alpha}{2(\alpha+2)}} m_0^{-1})^{\frac{2}{2\omega-1}}(-E(0))^{-(1-\frac{2}{(2\omega-1)(\alpha+2)})}$$
$$+ (\delta_1|\Omega|^{\frac{\alpha}{\alpha+2}} m_0^{-1})^{1/\omega}(-E(0))^{-(1-\frac{2}{\omega(\alpha+2)})}\Big\}.$$

p.26 ↓ 18 ~ p.27 ↓ 6 The proof of Theorem 2 should be changed into the following :

Claim B. If $E(0) < 0$ and $\alpha > \beta$, then

$$G(t)^{1/\omega} \le m_1 H(t) . \tag{2.6}$$

Indeed, Since $|(u', u)| \le B_2\|u'\|\|u\|_{\alpha+2}$ with $B_2 = |\Omega|^{\frac{\alpha}{2(\alpha+2)}}$, we have that

$$\begin{aligned} G(t)^{1/\omega} &\le 2\{(-E(t)) + (m_0^{-1}|P'(t)|)^{1/\omega}\} \\ &\le 2\{(-E(t)) + (2B_2 m_0^{-1}\|u'(t)\|\|u(t)\|_{\alpha+2} + \delta_1 B_2^2 m_0^{-1}\|u(t)\|_{\alpha+2}^2)^{1/\omega}\} \\ &\le 2\{(-E(t)) + 2\|u'(t)\|^2 + 2(2B_2 m_0^{-1}\|u(t)\|_{\alpha+2})^{2/(2\omega-1)} \\ &\quad + 2(\delta_1 B_2^2 m_0^{-1})^{1/\omega}\|u(t)\|_{\alpha+2}^{2/\omega}\}, \end{aligned}$$

where we used the Young inequality. Moreover, since $2/\omega < 2/(2\omega-1) < \alpha+2$ and

$$(-E(0))^{-1/(\alpha+2)}\|u(t)\|_{\alpha+2} \geq 1 \quad \text{if} \quad E(0) < 0$$

(see (2.2)), we observe that

$$\begin{aligned} G(t)^{1/\omega} \leq 2\{&(-E(t)) + 2\|u'(t)\|^2 \\ &+ 2(2B_2 m_0^{-1})^{\frac{2}{2\omega-1}}(-E(0))^{-(1-\frac{2}{(2\omega-1)(\alpha+2)})}\|u(t)\|_{\alpha+2}^{\alpha+2} \\ &+ 2(\delta_1 B_2^2 m_0^{-1})^{1/\omega}(-E(0))^{-(1-\frac{2}{\omega(\alpha+2)})}\|u(t)\|_{\alpha+2}^{\alpha+2}\}, \end{aligned}$$

and hence, we obtain (2.6).

p.40 ↓ 11 "bounded" → "non-increasing"

p.41 ↓ 6 ∼ ↓ 18 The proof of Theorem 9 should be changed into the following :

To proceed the estimation of (5.27), we observe from (1.6) and (5.2) that

$$\begin{aligned} |\delta_2(g(u'),u)| &\leq \delta_2\|u\|_{\beta+2}\|u'\|_{\beta+2}^{\beta+1} \\ &\leq \delta_2 c_*\|A^{1/2}u\|\|u'\|_{\beta+2}^{\beta+1}, \quad \beta \leq 4/(N-2) \\ &= \delta_2 c_*\|A^{1/2}u\|^{\frac{\beta}{\beta+2}}\|A^{1/2}u\|^{\frac{2}{\beta+2}}\|u'\|_{\beta+2}^{\beta+1} \\ &\leq \delta_2 c_*(d_*E(0))^{\frac{\beta}{2(\beta+2)}}\|u'\|_{\beta+2}^{\beta+1}\|A^{1/2}u\|^{\frac{2}{\beta+2}} \\ &\leq \frac{\beta+1}{\beta+2}(2\delta_2 c_*(d_*E(0))^{\frac{\beta}{2(\beta+2)}})^{\frac{\beta+2}{\beta+1}}\|u'\|_{\beta+2}^{\beta+2} + \frac{1}{2(\beta+2)}\|A^{1/2}u\|^2 \\ &\leq (2\delta_2 c_* d_*)^{\frac{\beta+2}{\beta+1}}\|u'\|_{\beta+2}^{\beta+2} + (1/4)\|A^{1/2}u\|^2, \end{aligned}$$

and hence, from $2K(u) \geq \|A^{1/2}u\|^2$,

$$\begin{aligned} \partial_t E^*(t) &\leq -2(\delta_1 + c_*^{-2}\delta_3 - \varepsilon)\|u'(t)\|^2 - (\varepsilon/2)\|A^{1/2}u(t)\|^2 \\ &\quad - (2\delta_2 - \varepsilon(2\delta_2 c_* d_*)^{\frac{\beta+2}{\beta+1}})\|u'(t)\|_{\beta+2}^{\beta+2} \\ &\leq -(\varepsilon/2)(\|u'(t)\|^2 + \|A^{1/2}u(t)\|^2), \end{aligned} \tag{5.31}$$

where we used (5.15) and we put

$$\varepsilon = \min\{(\delta_1 + c_*^{-2}\delta_3)/2,\, 2\delta_2(2\delta_2 c_* d_*)^{-\frac{\beta+2}{\beta+1}},\, (2d_*(c_* + c_*^2\delta_1 + \delta_3))^{-1}\}.$$