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SEMIGROUPS OF ORDER $\leqq$ 10 WHOSE GREATEST C-HOMOMORPHIC IMAGES ARE GROUPS

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In the present note we show all the isomorphically and anti-isomorphically distinct semigroups of order $\leqq 10$ whose greatest commutative homomorphic[1)] images are groups. Especially a semigroup which has no proper commutative homomorphic image is called c-indecomposable. The purpose of the computation is to obtain examples by which we test and clarify the theory of finite semigroups of this kind. Here we shall only show tables of the results, the theory being discussed precisely in publication elsewhere.

1. On Tables 1 and 2. Since a finite simple semigroup is completely simple, it is represented as a regular matrix semigroup, which is determined by a ground group G (or G_0 with zero) and a defining matrix P [1].

At first we shall explain the notations of Table 1 with examples.

$\lambda.\ \mu$		the μ-th simple semigroup of order λ
3. 1	$\{\varepsilon\}$. 3—1	the first simple semigroup of order 3 with the ground group $G = \{\varepsilon\}$ and the defining matrix $P = \begin{pmatrix}\varepsilon\\ \varepsilon\\ \varepsilon\end{pmatrix}$.
4. 3.	$\{\varepsilon,\ \alpha\}$, 2—1	the ground group : $G = \{\varepsilon,\ \alpha\}$, $\alpha^2 = \varepsilon$, the defining matrix : $P = \begin{pmatrix}\varepsilon\\ \varepsilon\end{pmatrix}$.
6. 4	$\{\varepsilon, \alpha, \beta\}$, 2—1	$G = \{\varepsilon, \alpha, \beta\}$, $\beta = \alpha^2$, $\alpha^3 = \varepsilon$, $P = \begin{pmatrix}\varepsilon\\ \varepsilon\end{pmatrix}$.
4. 2	$\{\varepsilon\}$, 2—2	$G = \{\varepsilon\}$, $P = \begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & \varepsilon\end{pmatrix}$.

Remark. $m - l$ symbols the matrix with m-rows l-columns, all the elements of which are ε.

5. 3	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$	$G_0 = \{0, \varepsilon\}$, $P = \begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$. Of course S has zero 0.
r-sing.		right-singular semigroup i. e. $xy = y$ for every x, y.
3. 1′		the dual form of the semigroup 3. 1, i. e. the multiplication $x \cdot y$ of 3. 1′ is defined as $x \cdot y = yx$ where yx is the multplication of 3. 1

1) "commutative homomorphic" will be called "c-homomorphic".

2.2×2.1	the direct product of the semigroups 2.2 and 2.1
c-ind.	c-indecomposable
c-dec.	c-decomposable
comm.	commutative
ind.	indecomposable i. e. having no proper homomorphism
self-dual	anti-isomorphic to itself.
	One unfilled in the column of "self-dual or not" is not self-dual.

Remark. For example, the semigroup 4. 3, $\{\varepsilon, \alpha\}$, 2—1, is composed of the elements

$$(\varepsilon,\ 11), \quad (\alpha,\ 11), \quad (\varepsilon,\ 12), \quad (\alpha,\ 12)$$

which are denoted by a, b, c, d respectively in the alphabet order ; 4. 3 represents

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	a	b	c	d
d	b	a	d	c

The semigroup 4. 2. $\{\varepsilon\}$, 2—2, is composed of

$$a=(\varepsilon,\ 11), \quad b=(\varepsilon,\ 12), \quad c=(\varepsilon,\ 21), \quad d=(\varepsilon,\ 22)$$

with the table

	a	b	c	d
a	a	b	a	b
b	a	b	a	b
c	c	d	c	d
d	c	d	c	d

The semigroup 5. 3, $\{0,\ \varepsilon\}$, $\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$ shows

	a	b	c	d	e
a	a	a	a	a	a
b	a	b	c	b	c
c	a	b	c	a	a
d	a	d	e	d	e
e	a	d	e	a	a

where $a=0$, $b=(\varepsilon,\ 11)$, $c=(\varepsilon,\ 12)$, $d=(\varepsilon,\ 21)$, $e=(\varepsilon,\ 22)$.

The semigroup 7. 3 $\{0,\ \varepsilon\}$, $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & 0\end{pmatrix}$ consists of the elements

$a=0$, $b=(\varepsilon,\ 11)$, $c=(\varepsilon,\ 12)$, $d=(\varepsilon,\ 21)$, $e=(\varepsilon,\ 22)$, $f=(\varepsilon,\ 31)$, $g=(\varepsilon,\ 32)$.

The semigroup 9. 13 $\{0, \varepsilon, \alpha\}$, $\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$ consists of the elements $a=0$, $b=(\varepsilon,\ 11)$, $c=(\alpha,\ 11)$, $d=(\varepsilon,\ 12)$, $e=(\alpha,\ 12)$, $f=(\varepsilon,\ 21)$, $g=(\alpha,\ 21)$, $h=(\varepsilon,\ 22)$, $i=(\alpha,\ 22)$.

In Table 2, there are given automorphisms of some simple semgroups and some non-simple semigroups. The table shows that, for example, the automorphisms of the semigroup 3. 2 are

$$\begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}, \qquad \begin{pmatrix} a & b & c \\ a & c & b \end{pmatrix},$$

and those of 5. 4 are

$$\begin{pmatrix} 0 & 11 & 12 & 21 & 22 \\ 0 & 11 & 12 & 21 & 22 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 11 & 12 & 21 & 22 \\ 0 & 22 & 21 & 12 & 11 \end{pmatrix}$$

which are also denoted by $\begin{pmatrix} a & b & c & d & e \\ a & b & c & d & e \end{pmatrix}$, $\begin{pmatrix} a & b & c & d & e \\ a & e & d & c & b \end{pmatrix}$ respectively.

With respect to 7. 3, there is an automorphism

$$\begin{pmatrix} 0 & 11 & 12 & 21 & 22 & 31 & 32 \\ 0 & 21 & 22 & 11 & 12 & 31 & 32 \end{pmatrix}$$

besides the identical mapping. The automorphisms are useful for us to exclude isomorphic semigroups in our computation.

2. General Remark. An ideal I[2] of a semigroup S is called proper if I is neither S itself nor an ideal composed of only zero. A proper ideal I of S is called minimal, if I contains no proper ideal of S i. e. $\{0\} \subset J \subset I$ for no ideal J of S. If S is finite and not simple, then a minimal ideal exists. It is known that a minimal ideal of a finite semigroup is either a simple semigroup or a semigroup defined as $xy = 0$ for all x, y. [2] The latter will be called zero-semigroup. Since a homomorphic image of a c-indecomposable semigroup is also c-indecomposable, the difference semigroup $D = (S: I)$ of a c-indecomposable semigroup S modulo an ideal I is c-indecomposable. Further if I is minimal and simple, then I is also c-indecomposable.

Our computation is to find all c-indecomposable semigroups S such that $D = (S: I)$ and I is a minimal ideal of S when I (simple c-indecomposable semigroup or a zero-semigroup) and a c-indecomposable semigroup D with zero are given. In particular the Tables $3 \sim 11$ show the cases where D is moreover simple.

When I is not a zero-remigroup, S is completely determined by a system of some right translations φ of I:

$$\Phi = \{\varphi_\alpha \mid \alpha \in D, \quad \alpha \neq 0\},$$

and a system of some left translations ψ of I:

$$\Psi = \{\psi_\alpha \mid \alpha \in D, \quad \alpha \neq 0\}$$

where the correspondence $\alpha \rightarrow \varphi_\alpha$ is a ramified homorphism of D and $\alpha \rightarrow \psi_\alpha$ is a ramified anti-homomorphis of D. Let f_a and g_a be an inner right translation of I and an inner left translation of I respectively:

$$f_a(x) = xa \qquad\qquad a \in I \qquad x \in I$$
$$g_a(x) = ax.$$

I is called right-regular if the correspondence $a \rightarrow \varphi_a$ is one-to-one; left-regularity

2) By an ideal we mean a two sided ideal.

is defined dually.
Especially if I is right-regular, S is completely determined by only the system Φ.

When I is a zero-semigroup, S is not always determined by Φ and Ψ : and then there is necessity for giving adequately the product of certain elements in order to determine S uniquely. We note that we may adopt $\{\varphi_\alpha \mid \alpha \in B \subset D, \alpha \neq 0\}$ instead of Φ, $\{\psi_\alpha \mid \alpha \in B \subset D, \alpha \neq 0\}$ instead of Ψ, where B is called the base of D.

Next, let S be a finite semigroup (c. d. g.) whose greatest c-homomorphic image G is a group, and let I be a minimal ideal of S. Then I is a simple semigroup without zero, and G is the greatest c-homomorphic image of I under the mapping $S \rightarrow G$, and further the difference semigroup $D = (S: I)$ is c-indecomposable. When there are given a simple semigroup (c. d. g.) I and a c-indecomposable semigroup D with zero, we must find S such that $D = (S: I)$ where I is a minimal ideal of S. The method of computation is like the case of c-indecomposable S.

3. On Contents of Tables. Generally $I — D$ symbols the type of a semigroup S such that I is a minimal ideal of S and $D = (S: I)$ is simple. $I_1 — \tilde{I}_2 — D'$ symbols that I_1 is a minimal ideal of S and the difference semigroup $D = (S: I_1)$ is not simple but have type $\tilde{I}_2 — D'$. In other words there is an ideal I_2 such that

$$(S: I_2) = D', \qquad (I_2: I_1) = \tilde{I}_2$$

where D' is simple. By $I_1 — \tilde{I}_2 — \tilde{I}_3 — D''$ we mean that $(S: I_1)$ is of type $\tilde{I}_2 — \tilde{I}_3 — D''$, namely there is a sequence of the ideals $I_1 \subset I_2 \subset I_3 \subset S$ where $(S: I_3) = D''$, $(I_3: I_2) = \tilde{I}_3$, $(I_2: I_1) = \tilde{I}_2$, and D'' is simple.

See Contents of Tables.

(r-sing.) — 5	The type in which I is right-singular and D is a simple semigroup of order 5 with zero i. e. 5.3 or 5.4.
5 (simp. 0)	A simple semigroup of order 5 with zero, 5.3 or 5.4.
5 (simp. 0) — 5	$I = 5$ (simp. 0), and D is also 5 (simp. 0)
(z) — 5	I is a zero-semigroup and D is same as the above.
3_0 — 5	I is the zero-semigroup of order 3, and D same as the above.
(sing.) — 7	I is singular, that is, right-singular or left-singular, and $D = 7$ (simp. 0) i. e. one of 7.3 ~ 7.6.
2 (sing.) — 9_ε	I is a singular semigroup of order 2 i. e. 2.1 or 2.1′, and D is one of 9.6 ~ 9.12.
2 (r-sing.) — $9_{\varepsilon,\alpha}$	$I = 2.1$, and D is either 9.13 or 9.14.
4 (r-sing. × l-sing.) — 7	I is of order 4 and the direct product of a right-singular semigroup and a left-singular semigroup.

3_0—3_0—5	I_1 is the zero-semigroup of order 3, and D has type 3_0—5; i. e. $\tilde{I}_2 = 3_0$, $D' = 5$ (simp. 0)
2 (r-sing.) —3_0—3_0—5	$I_1 = 2.1$ and D has type 3_0—3_0—5; $\tilde{I}_2 = 3_0$, $\tilde{I}_3 = 3_0$.
c. d. g.	c-decomposable and its greatest c-homomorphic image is a group.
(simp. or g.)	Either a group or the direct product of a group and a singular semigroup.
2_g	the group of order 2.

4. On Tables of the Non-simple. See Table 3, 3. 1—5. 3. We find three isomorphically distinct semigroups S denoted by (3. 1—5. 3) 1, (3. 1—5. 3) 2, (3. 1—5. 3) 3 :

(3. 1—5. 3) 1	$\varphi_{11} = (a\ a\ a)$	$\varphi_{22} = (a\ a\ a)$,
(3. 1—5. 3) 2	$\varphi_{11} = (b\ b\ b)$	$\varphi_{22} = (a\ a\ a)$,
(3. 1—5. 3) 3	$\varphi_{11} = (a\ c\ c)$	$\varphi_{22} = (a\ a\ b)$,

where 11, 22 form the base of D. The Tables show φ or ψ for the base of D.

As far as (3. 1—5. 3) 3 is concerned, we get

$$\varphi_{12} = \varphi_{11}\ \varphi_{22} = \begin{pmatrix} a & b & c \\ a & c & c \end{pmatrix} \begin{pmatrix} a & b & c \\ a & a & b \end{pmatrix} = \begin{pmatrix} a & b & c \\ a & b & b \end{pmatrix}, \quad \varphi_{21} = \varphi_{22}\ \varphi_{11} = \begin{pmatrix} a & b & c \\ a & a & c \end{pmatrix}$$

and

		a	b	c	11 d	12 e	21 f	22 g
	a	a	b	c	a	a	a	a
	b	a	b	c	c	b	a	a
	c	a	b	c	c	b	c	b
11	d	a	b	c	d	e	d	e
12	e	a	b	c	d	e	a	a
21	f	a	b	c	f	g	f	g
22	g	a	b	c	f	g	a	a

where, if, for example, we set $x = ef \in I$, then $\varphi_x = \varphi_{12}\ \varphi_{21} = (aaa)$ implies $x = a$ because I is right regular; the others are likewise found. Tables 3, 4, 6, 8, and 9 are seen in the same manner as this.

In Table 4, we seem that there is only one belonging type 5. 4—5. 3, but (5. 3—5. 4) 1 may be admitted into the category 5. 4—5. 3.

See Table 5. When I is 3_0 and D is 5. 3, the reguired $S = \{a, b, c, d, e, f, g\}$ is completely determined by φ_{11} and φ_{22} because we can prove that $\varphi_{11} = (a\,c\,c)$ and $\varphi_{22} = (a\,a\,b)$ imply $xy = a$ for $x = d, e, f, g$ and $y = a, b, c,$ and moreover we get $g^2 = (22)^2 = a$ and hence $uv = a$ if $uv \in I$, $u \in S$, $v \in S$. Similarly we have 3_0—5. 4, 5_0—5. 4 in Table 5, and 3_0—7, 4_0—7 in Table 7. In these cases, $xy \in I$, $x \bar{\in} I$, $y \bar{\in} I$ implies $xy = a$.

On the other hand, even if φ_{11}, φ_{22}, ψ_{11}, ψ_{22} are assigned, $S = \{a, b, c, d, e, f, g, h, i\}$ of type 5_0—5. 3 is not uniquely determined, but we have (5_0—

5. 3) 1 or (5_0—5. 3) 2 according as $i^2=(22)^2=a$ or b.
We add that if i^2 is given, every $xy \in I$ ($x \bar{\in} I$, $y \bar{\in} I$) is naturally determined :

$$gh=gi=ih=i^2=a \qquad \text{in } (5_0\text{—}5.3)\ 1,$$
$$gh=e,\quad gi=c,\quad ih=d,\quad i^2=b \qquad \text{in } (5_0\text{—}5.3)\ 2.$$

See Tables 10, and 11, For example, (4. 2—5. 3) 3 is completely determined by φ_e, φ_h, ψ_e, ψ_h. In fact we calculate

$$\varphi_f=\varphi_e\varphi_h=(aacc), \qquad \varphi_g=\varphi_h\varphi_e=(bbdd),$$
$$\psi_f=\psi_h\psi_e=(cdcd), \qquad \psi_g=\psi_e\psi_h=(abab),$$

and $\varphi_{xy}=\varphi_x\varphi_y$, $\psi_{xy}=\psi_y\psi_x$ for $x \bar{\in} I$, $y \bar{\in} I$, $xy \in I$
from which all xy are uniquely determined :

$$h^2=a,\quad hg=b,\quad fh=c,\quad fg=d.$$

In Tables 2 ~ 11, thus, we have seen the type I—D, while there are the type I_1—$\tilde{I}_2$—D' in Tables 12 ~ 17, 23 ~ 26, and the types I_1—$\tilde{I}_2$—$\tilde{I}_3$—D'' in Tables 18 and 27.

In Tables 12 and 13, I_2 is denoted by $\{abcaa\}$, $\{aaabc\}$ etc., which represent

	a	b	c	d	e
a	a	b	c	a	a
b	a	b	c	a	a
c	a	b	c	a	a
d	a	b	c	a	a
e	a	b	c	a	a

,

	a	b	c	d	e
a	a	a	a	b	c
b	a	a	a	b	c
c	a	a	a	b	c
d	a	a	a	b	c
e	a	a	a	b	c

etc.

respectively. The automorphisms of I_2 have already listed in Table 2. φ and ψ shown in Tables 12, 13 are right translations and left translations of I_1. We note that I_2 cannot be prepared arbitrarily, but is somewhat restricted by I_1, φ and ψ.

In Table 14, I_2 is denoted by

$$\begin{pmatrix} \varphi_e=(aacc) \\ \varphi_f=(aacc) \\ \psi_e=(abab) \\ \psi_f=(abab) \end{pmatrix}.$$

This represents

	a	b	c	d	e	f
a	a	b	a	b	a	a
b	a	b	a	b	a	a
c	c	d	c	d	c	c
d	c	d	c	d	c	c
e	a	b	a	b	a	a
f	a	b	a	b	a	a

which is obtained like (4. 2—5. 3).

Referring to the examples which we have explained, all the tables can be understood.

References

[1] D. Rees, On semigroups, Proc. Cambridge Philos. Soc., 36, 1940 387—400.
[2] S. Schwarz, On semigroup having a kernel, Czechoslovak Math. Jour., 1 76, 1951 25—88.

Contents of Tables

Contents by Classification of the Orders

Table 1 Simple Semigroups whose greatest c-homomorphic images are groups

Order	No.	defining matrix	Remark	c-decomposability	self-dual or not
2	2.1	$\{\varepsilon\}$, 2-1	r-sing.	c-ind.	
	2.2		group	comm.	
3	3.1	$\{\varepsilon\}$, 3-1	r-sing.	c-ind.	
	3.2		group	comm.	
4	4.1	$\{\varepsilon\}$, 4-1	r-sing.	c-ind.	
	4.2	$\{\varepsilon\}$, 2-2	$2.1\times 2.1'$	c-ind.	self-dual
	4.3	$\{\varepsilon, \alpha\}$, 2-1	2.2×2.1	c-dec.	
	4.4		cyclic group	comm.	
	4.5		group 2.2×2.2	comm.	
5	5.1	$\{\varepsilon\}$, 5-1	r-sing.	c-ind.	
	5.2		group	comm.	
	5.3	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\\ \varepsilon 0\end{pmatrix}$		ind.	self-dual
	5.4	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon 0\\ 0\varepsilon\end{pmatrix}$		ind.	self-dual
6	6.1	$\{\varepsilon\}$, 6-1	r-sing.	c-ind.	
	6.2	$\{\varepsilon\}$, 2-3	$2.1\times 3.1'$	c-ind.	
	6.3	$\{\varepsilon, \alpha\}$, 3-1	2.2×3.1	c-dec.	
	6.4	$\{\varepsilon, \alpha, \beta\}$, 2-1	3.2×2.1	c-dec.	
	6.5		cyclic group	c-dec.	
	6.6		symmetric group	c-dec.	
7	7.1	$\{\varepsilon\}$, 7-1	r-sing.	c-ind.	
	7.2		group	comm.	
	7.3	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon 0\end{pmatrix}$		c-ind.	
	7.4	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon 00\end{pmatrix}$		c-ind.	
	7.5	$\{0, \varepsilon\}$, $\begin{pmatrix}0\varepsilon 0\\ \varepsilon 0\varepsilon\end{pmatrix}$		c-ind.	
	7.6	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\end{pmatrix}$		ind.	
8	8.1	$\{\varepsilon\}$, 8-1	r-sing.	c-ind.	
	8.2	$\{\varepsilon\}$, 4-2	$4.1\times 2.1'$	c-ind.	
	8.3	$\{\varepsilon, \alpha\}$, 2-2	2.2×4.2	c-dec.	self-dual
	8.4	$\{\varepsilon, \alpha\}$, $\begin{pmatrix}\varepsilon\varepsilon\\ \varepsilon\alpha\end{pmatrix}$		c-ind.	self-dual
	8.5	$\{\varepsilon, \alpha\}$, 4-1	2.2×4.1	c-dec.	
	8.6		cyclic group	comm.	
	8.7		group 2.2×4.4	comm.	
	8.8		group 2.2×4.5	comm.	
	8.9		dihedral group	c-dec.	
	8.10		quaternion group	c-dec.	
9	9.1	$\{\varepsilon\}$, 9-1	r-sing,	c-ind.	
	9.2	$\{\varepsilon\}$, 3-3	$3.1\times 3.1'$	c-ind.	
	9.3	$\{\varepsilon, \alpha, \beta\}$, 3-1	3.2×3.1	c-dec.	
	9.4		cyclic group	comm.	
	9.5		group 3.2×3.2	comm.	
	9.6	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon\varepsilon 0\end{pmatrix}$		c-ind.	
	9.7	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon 00\end{pmatrix}$		c-ind.	
	9.8	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon 0\\ \varepsilon\varepsilon 0\varepsilon\end{pmatrix}$		c-ind.	
	9.9	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\varepsilon\\ \varepsilon 000\end{pmatrix}$		c-ind.	
	9.10	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 00\\ \varepsilon 0\varepsilon\varepsilon\end{pmatrix}$		c-ind.	
	9.11	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon 0\\ 000\varepsilon\end{pmatrix}$		c-ind.	
	9.12	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 00\\ 00\varepsilon\varepsilon\end{pmatrix}$		c-ind.	
	9.13	$\{0, \varepsilon, \alpha\}$, $\begin{pmatrix}\varepsilon\varepsilon\\ \varepsilon 0\end{pmatrix}$		c-ind.	self-dual
	9.14	$\{0, \varepsilon, \alpha\}$, $\begin{pmatrix}\varepsilon 0\\ 0\varepsilon\end{pmatrix}$		c-ind.	self-dual

	10. 1	$\{\varepsilon\}$, 10-1	r-sing.	c-ind.	
	10. 2	$\{\varepsilon\}$, 2-5	2. 1×5. 1′	c-ind.	
	10. 3	$\{\varepsilon, \alpha\}$, 5-1	2. 2×5. 1	c-dec.	
	10. 4	$\{\varepsilon, \alpha, \beta, \gamma, \delta,\}$ 2-1	5. 2×2. 1	c-dec.	
	10. 5		cyclic group	comm.	
	10. 6		non-commutative group	c-dec.	
	10. 7	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon 0\end{pmatrix}$		c-ind.	self-dual
	10. 8	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon\varepsilon\\ \varepsilon 00\end{pmatrix}$		c-ind.	
	10. 9	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\end{pmatrix}$		ind.	self-dual
	10. 10	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon\varepsilon 0\\ \varepsilon 00\end{pmatrix}$		ind.	self-dual
10	10. 11	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\end{pmatrix}$		ind.	
	10. 12	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\\ 0\varepsilon\varepsilon\end{pmatrix}$		ind.	self-dual
	10. 13	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon\varepsilon\\ \varepsilon 00\\ \varepsilon 00\end{pmatrix}$		c-ind.	self-dual
	10. 14	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\\ \varepsilon 00\end{pmatrix}$		ind.	
	10. 15	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon\varepsilon 0\\ 00\varepsilon\end{pmatrix}$		c-ind.	self-dual
	10. 16	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon\varepsilon 0\\ \varepsilon 0\varepsilon\\ 0\varepsilon 0\end{pmatrix}$		ind.	self-dual
	10. 17	$\{0, \varepsilon\}$, $\begin{pmatrix}00\varepsilon\\ \varepsilon\varepsilon 0\\ \varepsilon 00\end{pmatrix}$		ind.	
	10. 18	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon 00\\ \varepsilon 00\\ 0\varepsilon\varepsilon\end{pmatrix}$		c-ind.	
	10. 19	$\{0, \varepsilon\}$, $\begin{pmatrix}\varepsilon 00\\ 0\varepsilon 0\\ 00\varepsilon\end{pmatrix}$		ind.	self-dual

Table 2 Automorphisms

(sing.) (singular semigroup)	all permutations
(Z) ($xy = 0$ for all x, y)	all permutations which fix 0
2. 2 ························	*ab*
3. 2 ························	*abc, acb*
4. 2 ························	*abcd, badc, cdab, dcba*
4. 3 ························	*abcd, cdab*
4. 4 ························	*abcd, adcb*
4. 5 ························	*abcd, abdc, acbd, adcb*
5. 2 ························	*abcde, acebd, adbec, aedcb*
5. 3 ························	$\begin{matrix} a & b & c & d & e \\ 0 & 11 & 12 & 21 & 22 \end{matrix}$
5. 4 ························	$\begin{matrix} a & b & c & d & e, & a & e & d & c & b \\ 0 & 11 & 12 & 21 & 22 & 0 & 22 & 21 & 12 & 11 \end{matrix}$
6. 2 ·····················	*abcdef, badcfe, abefcd, bafedc,* *cdabef, dcbafe, cdefab, dcfeba,* *efabcd, febadc, efcdab, fedcba*
6. 3 ·····················	*abcdef, abefcd, cdabef, cdefab,* *efabcd, efcdab*

6.4 ……………………… *abcdef*, *acbdfe*, *defabc*, *dfeacb*

6.5 ……………………… *abcdef*, *afedcb*

6.6 ……………………… { *abcdef*, *abcefd*, *abcfde*, *acbdfe*, *acbedf*, *acbfed* }

7.3 ……………………… 0 11 12 21 22 31 32, 0 21 22 11 12 31 32

7.4 ……………………… 0 11 12 21 22 31 32. 0 11 12 31 32 21 22

7.5 ……………………… 0 11 12 21 22 31 32, 0 31 32 21 22 11 12

7.6 ……………………… 0 11 12 21 22 31 32, 0 12 11 32 31 22 21

9.6 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 31 32 21 22 41 42
0 21 22 11 12 31 32 41 42
0 21 22 31 32 11 12 41 42
0 31 32 11 12 21 22 41 42
0 31 32 21 22 11 12 41 42

9.7 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 21 22 41 42 31 32
0 21 22 11 12 31 32 41 42
0 21 22 11 12 41 42 31 32

9.8 ………………………
0 11 12 21 22 31 32 41 42
0 21 22 11 12 31 32 41 42
0 12 11 22 21 42 41 32 31
0 22 21 12 11 42 41 32 31

9.9 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 21 22 41 42 31 32
0 11 12 31 32 21 22 41 42
0 11 12 31 32 41 42 21 22
0 11 12 41 42 21 22 31 32
0 11 12 41 42 31 32 21 22

9.10 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 21 22 41 42 31 32

9.11 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 31 32 21 22 41 42
0 21 22 11 12 31 32 41 42
0 21 22 31 32 11 12 41 42
0 31 32 11 12 21 22 41 42
0 31 32 21 22 11 12 41 42

9.12 ………………………
0 11 12 21 22 31 32 41 42
0 11 12 21 22 41 42 31 32
0 21 22 11 12 31 32 41 42
0 21 22 11 12 41 42 31 32
0 32 31 42 41 12 11 22 21
0 32 31 42 41 22 21 12 11
0 42 41 32 31 12 11 22 21
0 42 41 32 31 22 21 12 11

{*abaa*} ……………………… *abcd*, *abdc*

{*abab*} ……………………… *abcd*, *badc*

{*abcaa*} ……………………… *abcde*, *abced*, *acbde*, *acbed*

{*abcab*} ……………………… *abcde*, *baced*,

{*abcdaa*} ……………………… *axyzef*, *axyzfe* ($x, y, z = b, c, d$)

{*abcdab*} ……………………… *abcdef*, *abdcef*, *bacdfe*, *badcfe*

{*aaabc*} ……………………… *abcde*, *acbed*

3) $\left(4.2;\ \begin{matrix} \varphi_e = (aacc), & \varphi_f = (aacc) \\ \psi_e = (abab), & \psi_f = (abab) \end{matrix}\right)$ ………… *abcdef*, *abcdfe*

$\left(4.2;\ \begin{matrix} \varphi_e = (aacc), & \varphi_f = (bbdd) \\ \psi_e = (abab), & \psi_f = (abab) \end{matrix}\right)$ ………… *abcdef*, *badcfe*

(2.1—5.3) 1 ……………………… *abcdef*

(2.1—5.3) 2 ……………………… *abcdef*

(2.1—5.4) 1 ……………………… *abcdef*, *abfedc*

(2.1—5.4) 2 ……………………… *abcdef*, *bafedc*

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3) See p. 48, this paper.

$\{abaaaa\}$	$abxyzu$ $((x\ y\ z\ u)$ perm. of $c, d, e, f)$[4]
$\{abaabb\}$	$abcdef,\ abcdfe,\ abdcef,\ abdcfe,$ $baefcd,\ baefdc,\ bafecd,\ bafedc$
$\{abaaab\}$	$abxyzf$ $((x\ y\ z)$ perm. of $c, d, e)$
$\{abaaba\}$	$abxyez$ $((x\ y\ z)$ perm. of $c, d, f)$
$\{ababab\}$	$abcdef,\ abcfed,\ abedcf,\ abefcd,$ $badcfe,\ badefc,\ bafcde,\ bafedc$
$\{abaacd\}$	$abcdef,\ abdcfe$

4) $(x\ y\ z\ u)$ runs throughout the permutations of c, d, e, f.

Table 3

c-ind. (r-sing.) —5

I \ D	5.3 $\begin{pmatrix}\varepsilon & \varepsilon \\ \varepsilon & 0\end{pmatrix}$ No.	φ_{11}	φ_{22}	5.4 $\begin{pmatrix}\varepsilon & 0 \\ 0 & \varepsilon\end{pmatrix}$ No.	φ_{12}	φ_{21}
2.1	1	a a	a a	1	a a	a a
	2	a a	b b	2	a a	b b
3.1	1	a a a	a a a	1	a a a	a a a
	2	b b b	a a a	2	a a a	b b b
	3	a c c	a a b	8	a a b	a c a
				4	a a b	b c c
4.1	1	a a a a	a a a a	1	a a a a	a a a a
	2	b b b b	a a a a	2	a a a a	b b b b
	3	a d a d	a a a b	3	a a a b	a d a a
	4	a d d d	a a a b	4	a a a b	a d d a
	5	c d c d	a a a b	5	a a a b	b d b d
	6	a c c a	a a b b	6	a a a b	b d d d
	7	a c c c	a a b b	7	a a a b	c d c c
				8	a a a b	c d d d
				9	a a b b	a c a c
				10	a a b b	b c c b
5.1	1	a a a a a	a a a a a	1	a a a a a	a a a a a
	2	b b b b b	a a a a a	2	a a a a a	b b b b b
	3	a e a a e	a a a a b	3	a a a a b	a e a a a
	4	a e a e e	a a a a b	4	a a a a b	a e a e a
	5	a e e e e	a a a a b	5	a a a a b	a e e e a
	6	c e c c e	a a a a b	6	a a a a b	b e b b e
	7	c e c e e	a a a a b	7	a a a a b	b e b e e
	8	a d a d a	a a a b b	8	a a a a b	b e e e e
	9	a d a d d	a a a b b	9	a a a a b	c e c c c
	10	a d d d a	a a a b b	10	a a a a b	c e c e c
	11	a d d d d	a a a b b	11	a a a a b	c e e c e
	12	c d c d c	a a a b b	12	a a a a b	c e e e e
	13	c d c d d	a a a b b	13	a a a b b	a d a a d
	14	a c c a a	a a b b b	14	a a a b b	a d d a a
	15	a c c a c	a a b b b	15	a a a b b	a d d a d
	16	a c c c c	a a b b b	16	a a a b b	b d b d b
	17	a d e d e	a a a b c	17	a a a b b	b d b d d
				18	a a a b b	b d d d b
				19	a a a b b	c d c c d
				20	a a a b b	c d d d c
				21	a a b b b	a c a c c
				22	a a b b b	b c c b b
				23	a a a b c	a d e a a
				24	a a a b c	b d e d d
				25	a a a b c	c d e e e
6.1	1	a a a a a a	a a a a a a	1	a a a a a a	a a a a a a
	2	b b b b b b	a a a a a a	2	a a a a a a	b b b b b b
	3	a f a a a f	a a a a a b	3	a a a a a b	a f a a a a
	4	a f a a f f	a a a a a b	4	a a a a a b	a f a a f a
	5	a f a f f f	a a a a a b	5	a a a a a b	a f a f f a
	6	a f f f f f	a a a a a b	6	a a a a a b	a f f f f a
	7	c f c c c f	a a a a a b	7	a a a a a b	b f b b b f

	8	c f c c f f	a a a a a b	8	a a a a a b	b f b b f f
	9	c f c f f f	a a a a a b	9	a a a a a b	b f b f f f
	10	a e a a e a	a a a a b b	10	a a a a a b	b f f f f f
	11	a e a e e a	a a a a b b	11	a a a a a b	c f c c c c
	12	a e e e e a	a a a a b b	12	a a a a a b	c f c c f c
	13	a e a a e e	a a a a b b	13	a a a a a b	c f c f f c
	14	a e a e e e	a a a a b b	14	a a a a a b	c f f c c f
	15	a e e e e e	a a a a b b	15	a a a a a b	c f f c f f
	16	c e c c e c	a a a a b b	16	a a a a a b	c f f f f f
	17	c e c c e e	a a a a b b	17	a a a a b b	a e a a a e
	18	c e c e e c	a a a a b b	18	a a a a b b	a e a e a a
	19	c e c e e e	a a a a b b	19	a a a a b b	a e a e a e
	20	a d a d a a	a a a b b b	20	a a a a b b	a e e e a a
	21	a d a d a d	a a a b b b	21	a a a a b b	a e e e a e
	22	a d a d d d	a a a b b b	22	a a a a b b	b e b b e b
	23	a d d d a a	a a a b b b	23	a a a a b b	b e b b e e
6.1	24	a d d d a d	a a a b b b	24	a a a a b b	b e b e e b
	25	a d d d d d	a a a b b b	25	a a a a b b	b e b e e e
	26	c d c d c c	a a a b b b	26	a a a a b b	b e e e e b
	27	c d c d c d	a a a b b b	27	a a a a b b	c e c c c e
	28	c d c d d d	a a a b b b	28	a a a a b b	c e c e c c
	29	a c c a a a	a a b b b b	29	a a a a b b	c e c e c e
	30	a c c a a c	a a b b b b	30	a a a a b b	c e e c e c
	31	a c c a c c	a a b b b b	31	a a a a b b	c e e c e e
	32	a c c c c c	a a b b b b	32	a a a a b b	c e e e e c
	33	a e f a e f	a a a a b c	33	a a a b b b	a d a a d d
	34	a e f e e f	a a a a b c	34	a a a b b b	a d d a a d
	35	a e f f e f	a a a a b c	35	a a a b b b	a d d a d d
	36	d e f d e f	a a a a b c	36	a a a b b b	b d b d b b
	37	a d f d a f	a a a b b c	37	a a a b b b	b d b d b d
	38	a d f d d f	a a a b b c	38	a a a b b b	b d d d b b
	39	a d f d f f	a a a b b c	39	a a a b b b	c d c c d d
				40	a a a b b b	c d d d c c
				41	a a b b b b	a c a c c c
				42	a a b b b b	b c c b b b
				43	a a a a b c	a e f a a a
				44	a a a a b c	a e f e a a
				45	a a a a b c	a e f f a a
				46	a a a a b c	b e f b e e
				47	a a a a b c	b e f e e e
				48	a a a a b c	b e f f e e
				49	a a a a b c	d e f d d d
				50	a a a a b c	d e f e e e
				51	a a a a b c	d e f f f f
				52	a a a b b c	a d f a d a
				53	a a a b b c	a d f a f a
				54	a a a b b c	b d f d b d
				55	a a a b b c	b d f d f d
				56	a a a b b c	c d f f c f
				57	a a a b b c	c d f f d f

Table 4 c-ind. 5 (simp. 0) — 5

D / I	5.3 $\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$			5.4 $\begin{pmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{pmatrix}$		
	No.	φ_{11}	φ_{22}	No.	φ_{12}	φ_{21}
5.3 $\begin{pmatrix}\varepsilon\varepsilon\\ \varepsilon 0\end{pmatrix}$	1	a a a a a	a a a a a	1	a a a a a	a a a a a
	2	a b b d d	a b b d d	2	a b b d d	a b b d d
	3	a b b d d	a c c e e	3	a b b d d	a c c e e
aaaaa	4	a b b d d	a b a d a	4	a b b d d	a b a d a
abcbc	5	a b b d d	a c a e a	5	a b b d d	a c a e a
abcaa	6	a b a d a	a b a d a	6	a b a d a	a b a d a
adede	7	a c c e e	a b b d d	7	a c c e e	a c c e e
adeaa	8	a c c e e	a c c e e			
	9	a b a d a	a b b d d			
5.4		a b a d a	a b a d a	1	a a a a a	a a a a a
aaaaa $\begin{pmatrix}\varepsilon 0\\ 0\varepsilon\end{pmatrix}$				2	a b a d a	a b a d a
abcaa				3	a a b a d	a c a e a
aaabc						
adeaa						
aaade						

Table 5

c-ind. (z) — 5

D / I	5.3 $\begin{pmatrix}\varepsilon\varepsilon\\ \varepsilon 0\end{pmatrix}$						5.4 $\begin{pmatrix}\varepsilon 0\\ 0\varepsilon\end{pmatrix}$				
	No.	φ_{11}	φ_{22}	ψ_{11}	ψ_{22}	$(22)^2$	φ_{12}	φ_{21}	ψ_{12}	ψ_{21}	$(12)^2=(21)^2$
3_0		*acc*	*aab*	*aaa*	*aaa*	*a*	*aab*	*aca*	*aaa*	*aaa*	*a*
5_0	1	*adede*	*aaabc*	*accee*	*aabad*	*a*	*aaabc*	*adeaa*	*acaea*	*aabad*	*a*
	2	*adede*	*aaabc*	*accee*	*aabad*	*b*					

Table 6

c-ind. (sing.) — 7

D / I	7.3 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & 0\end{pmatrix}$				7.4 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon\\ \varepsilon & 0 & 0\end{pmatrix}$			
	No.	φ_{11}	φ_{21}	φ_{32}	No.	φ_{11}	φ_{22}	φ_{31}
2.1	1	*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*
	2	*b b*	*b b*	*a a*	2	*b b*	*a a*	*b b*
3.1	1	*a a a*	*a a a*	*a a a*	1	*a a a*	*a a a*	*a a a*
	2	*b b b*	*b b b*	*a a a*	2	*b b b*	*a a a*	*b b b*
	3	*a c c*	*a c c*	*a a b*	3	*a c c*	*a a b*	*a a c*
4.1	1	*a a a a*	*a a a a*	*a a a a*	1	*a a a a*	*a a a a*	*a a a a*
	2	*b b b b*	*b b b b*	*a a a a*	2	*b b b b*	*a a a a*	*b b b b*
	3	*a d a d*	*a d a d*	*a a a b*	3	*a d a d*	*a a a b*	*a a a d*
	4	*a d a d*	*a d d d*	*a a a b*	4	*a d a d*	*a a a b*	*a a d d*
	5	*a d d d*	*a d d d*	*a a a b*	5	*a d d d*	*a a a b*	*a a a d*
	6	*c d c d*	*c d c d*	*a a a b*	6	*a d d d*	*a a a b*	*a a d d*
	7	*a c c a*	*a c c a*	*a a b b*	7	*c d c d*	*a a a b*	*c c c d*
	8	*a c c a*	*a c c c*	*a a b b*	8	*c d c d*	*a a a b*	*d d c d*
	9	*a c c c*	*a c c c*	*a a b b*	9	*a c c a*	*a a b b*	*a a c a*
					10	*a c c a*	*a a b b*	*a a c c*
					11	*a c c c*	*a a b b*	*a a c a*
					12	*a c c c*	*a a b b*	*a a c c*
	No.	ψ_{11}	ψ_{21}	ψ_{32}	No.	ψ_{11}	ψ_{22}	ψ_{31}
2.1′	1	*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*
	2	*a a*	*b b*	*a a*	2	*a a*	*a a*	*b b*
	3	*b b*	*b b*	*a a*	3	*b b*	*a a*	*a a*
					4	*b b*	*a a*	*b b*
3.1′	1	*a a a*	*a a a*	*a a a*	1	*a a a*	*a a a*	*a a a*
	2	*a a a*	*b b b*	*a a a*	2	*a a a*	*a a a*	*b b b*
	3	*b b b*	*b b b*	*a a a*	3	*b b b*	*a a a*	*a a a*
	4	*b b b*	*c c c*	*a a a*	4	*b b b*	*a a a*	*b b b*
	5	*a c c*	*a c c*	*a a b*	5	*b b b*	*a a a*	*c c c*
					6	*a c c*	*a a b*	*a b b*
4.1′	1	*a a a a*	*a a a a*	*a a a a*	1	*a a a a*	*a a a a*	*a a a a*
	2	*a a a a*	*b b b b*	*a a a a*	2	*a a a a*	*a a a a*	*b b b b*
	3	*b b b b*	*b b b b*	*a a a a*	3	*b b b b*	*a a a a*	*a a a a*
	4	*b b b b*	*c c c c*	*a a a a*	4	*b b b b*	*a a a a*	*b b b b*
	5	*a d a d*	*a d a d*	*a a a b*	5	*b b b b*	*a a a a*	*c c c c*
	6	*a d a d*	*c d c d*	*a a a b*	6	*a d a d*	*a a a b*	*a b a b*
	7	*a d d d*	*a d d d*	*a a a b*	7	*a d d d*	*a a a b*	*a b b b*
	8	*c d c d*	*c d c d*	*a a a b*	8	*a d d d*	*a a a b*	*a c c c*
	9	*a c c a*	*a c c a*	*a a b b*	9	*c d c d*	*a a a b*	*a b a b*
	10	*a c c c*	*a c c c*	*a a b b*	10	*c d c d*	*a a a b*	*c b c b*
					11	*a c c a*	*a a b b*	*a b b a*
					12	*a c c a*	*a a b b*	*d c c d*
					13	*a c c c*	*a a b b*	*a b b b*
D / I	7.5 $\begin{pmatrix}0 & \varepsilon & 0\\ \varepsilon & 0 & \varepsilon\end{pmatrix}$				7.6 $\begin{pmatrix}\varepsilon & \varepsilon & 0\\ \varepsilon & 0 & \varepsilon\end{pmatrix}$			
	No.	φ_{11}	φ_{22}	φ_{31}	No.	φ_{12}	φ_{21}	φ_{31}
2.1	1	*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*
	2	*b b*	*a a*	*b b*	2	*b b*	*a a*	*a a*

3. 1	1	a a a	a a a	a a a	1	a a a	a a a	a a a
	2	b b b	a a a	b b b	2	b b b	a a a	a a a
	3	a c a	a a b	a c a	3	a c c	a b a	a a b
4. 1	1	a a a a	a a a a	a a a a	1	a a a a	a a a a	a a a a
	2	b b b b	a a a a	b b b b	2	b b b b	a a a a	a a a a
	3	a d a a	a a a b	a d a a	3	a d a d	a b a a	a a a b
	4	a d d a	a a a b	a d d a	4	a d a d	a b b a	a a a b
	5	b d b d	a a a b	b d b d	5	a d d d	a b a a	a a a b
	6	b d d d	a a a b	b d b d	6	a d d d	a b b a	a a a b
	7	c d d d	a a a b	c d d d	7	c d c d	a b a a	a a a b
	8	a c a c	a a b b	a c a c	8	c d c d	a b b b	a a a b
					9	a c c a	a b a a	a a b b
					10	a c c a	a b a b	a a b b
					J1	a c c c	a b a a	a a b b
					12	a c c c	a b a b	a a b b
	No.	ψ_{11}	ψ_{22}	ψ_{31}	No.	ψ_{12}	ψ_{21}	ψ_{31}
2. 1′	1	a a	a a	a a	1	a a	a a	a a
	2	a a	a a	b b	2	a a	b b	a a
	3	b b	a a	b b	3	b b	a a	a a
					4	b b	b b	a a
3. 1′	1	a a a	a a a	a a a	1	a a a	a a a	a a a
	2	a a a	a a a	b b b	2	a a a	b b b	a a a
	3	b b b	a a a	b b b	3	b b b	a a a	a a a
	4	b b b	a a a	c c c	4	b b b	b b b	a a a
	5	a c a	a a b	a c a	5	b b b	c c c	a a a
	6	b c c	a a b	b c c	6	a c c	b b c	a a b
4. 1′	1	a a a a	a a a a	a a a a	1	a a a a	a a a a	a a a a
	2	a a a a	a a a a	b b b b	2	a a a a	b b b b	a a a a
	3	b b b b	a a a a	b b b b	3	b b b b	a a a a	a a a a
	4	b b b b	a a a a	c c c c	4	b b b b	b b b b	a a a a
	5	a d a a	a a a b	a d a a	5	b b b b	c c c c	a a a a
	6	a d d a	a a a b	a d d a	6	a d a d	b b b d	a a a b
	7	b d b d	a a a b	b d b d	7	a d d d	b b b d	a a a b
	8	b d d d	a a a b	b d d d	8	a d d d	c c c d	a a a b
	9	c d d d	a a a b	c d d d	9	c d c d	b b b d	a a a b
	10	c d c c	a a a b	c d c c	10	a c c a	a a d d	a a b b
	11	b d d d	a a a b	c d d d	11	a c c a	b b c c	a a b b
	12	c d c c	a a a b	a d a a	12	a c c c	b b c c	a a b b
	13	a c a a	a a b b	a c a a	13	a c c c	b b d d	a a b b
	14	a c a c	a a b b	a c a c				
	15	b d b d	a a b b	b d b d				
	16	b d d d	a a b b	b d d d				
	17	a d a a	a a b b	a c a a				
	18	a c a c	a a b b	b d b d				
	19	b c c c	a a b b	b d d d				

Table 7

c-ind. (z) — 7

D / I	7.3 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 0\end{pmatrix}$							7.4 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 0 & 0\end{pmatrix}$						
	No.	φ_{11}	φ_{21}	φ_{32}	ψ_{11}	ψ_{21}	ψ_{32}	No.	φ_{11}	φ_{22}	φ_{31}	ψ_{11}	ψ_{22}	ψ_{31}
3_0	1	acc	acc	aab	aaa	aaa	aaa	1	acc	aab	aac	aaa	aaa	aaa
	2	aaa	aaa	aaa	acc	acc	aab	2	aaa	aaa	aaa	acc	aab	abb
4_0		aaaa	aaaa	aaaa	addd	accc	aabb		aaaa	aaaa	aaaa	addd	aaac	abbb

D / I	7.5 $\begin{pmatrix}0 & \varepsilon & 0 \\ \varepsilon & 0 & \varepsilon\end{pmatrix}$							7.6 $\begin{pmatrix}\varepsilon & \varepsilon & 0 \\ \varepsilon & 0 & \varepsilon\end{pmatrix}$						
	No.	φ_{11}	φ_{22}	φ_{31}	ψ_{11}	ψ_{22}	ψ_{31}		φ_{12}	φ_{21}	φ_{31}	ψ_{12}	ψ_{21}	ψ_{31}
3_0	1	aca	aca	aab	aaa	aaa	aaa		acc	aba	aab	aaa	aaa	aaa
	2	aaa	aaa	aaa	aab	aca	aab							
4_0		aaaa	aaaa	aaaa	aada	acac	aaba		aaaa	aaaa	aaaa	adad	aacc	aabb

Table 8

c-ind. 2 (sing.) — 9_{ε}

I \ D	9.6 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 0\end{pmatrix}$					9.7 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 0 & 0\end{pmatrix}$				
	No.	φ_{11}	φ_{21}	φ_{31}	φ_{42}	No.	φ_{11}	φ_{22}	φ_{32}	φ_{42}
2.1	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$a\ a$	$a\ a$	$b\ b$	2	$a\ a$	$b\ b$	$b\ b$	$b\ b$
	No.	ψ_{11}	ψ_{21}	ψ_{31}	ψ_{42}	No.	ψ_{11}	ψ_{22}	ψ_{32}	ψ_{42}
2.1′	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$a\ a$	$a\ a$	$b\ b$	2	$a\ a$	$b\ b$	$a\ a$	$a\ a$
	3	$a\ a$	$a\ a$	$b\ b$	$a\ a$	3	$b\ b$	$a\ a$	$a\ a$	$a\ a$
	4	$a\ a$	$a\ a$	$b\ b$	$b\ b$	4	$b\ b$	$b\ b$	$a\ a$	$a\ a$
						5	$a\ a$	$a\ a$	$a\ a$	$b\ b$
						6	$a\ a$	$b\ b$	$a\ a$	$b\ b$
						7	$b\ b$	$a\ a$	$a\ a$	$b\ b$
						8	$b\ b$	$b\ b$	$a\ a$	$b\ b$

I \ D	9.8 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon & 0 \\ \varepsilon & \varepsilon & 0 & \varepsilon\end{pmatrix}$					9.9 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 0 & 0 & 0\end{pmatrix}$				
	No.	φ_{11}	φ_{21}	φ_{32}	φ_{41}	No.	φ_{11}	φ_{22}	φ_{32}	φ_{42}
2.1	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$a\ a$	$b\ b$	$a\ a$	2	$a\ a$	$b\ b$	$b\ b$	$b\ b$
	No.	ψ_{11}	ψ_{21}	ψ_{32}	ψ_{41}	No.	ψ_{11}	ψ_{22}	ψ_{32}	ψ_{42}
2.1′	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$a\ a$	$a\ a$	$b\ b$	2	$b\ b$	$a\ a$	$a\ a$	$a\ a$
	3	$a\ a$	$a\ a$	$b\ b$	$a\ a$	3	$a\ a$	$a\ a$	$a\ a$	$b\ b$
	4	$a\ a$	$a\ a$	$b\ b$	$b\ b$	4	$b\ b$	$a\ a$	$a\ a$	$b\ b$
	5	$a\ a$	$b\ b$	$a\ a$	$a\ a$					
	6	$a\ a$	$b\ b$	$a\ a$	$b\ b$					
	7	$a\ a$	$b\ b$	$b\ b$	$a\ a$					
	8	$a\ a$	$b\ b$	$b\ b$	$b\ b$					

I \ D	9.10 $\begin{pmatrix}\varepsilon & \varepsilon & 0 & 0 \\ \varepsilon & 0 & \varepsilon & \varepsilon\end{pmatrix}$					9.11 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon & 0 \\ 0 & 0 & 0 & \varepsilon\end{pmatrix}$				
	No.	φ_{12}	φ_{21}	φ_{31}	φ_{41}	No.	φ_{12}	φ_{22}	φ_{32}	φ_{41}
2.1	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$b\ b$	$a\ a$	$a\ a$	$a\ a$	2	$a\ a$	$a\ a$	$a\ a$	$b\ b$
	No.	ψ_{12}	ψ_{21}	ψ_{31}	ψ_{41}	No.	ψ_{12}	ψ_{22}	ψ_{32}	ψ_{41}
2.1′	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$a\ a$	$b\ b$	$a\ a$	2	$a\ a$	$a\ a$	$a\ a$	$b\ b$
	3	$a\ a$	$b\ b$	$a\ a$	$a\ a$	3	$a\ a$	$a\ a$	$b\ b$	$a\ a$
	4	$a\ a$	$b\ b$	$b\ b$	$a\ a$	4	$a\ a$	$a\ a$	$b\ b$	$b\ b$
	5	$b\ b$	$a\ a$	$a\ a$	$a\ a$					
	6	$b\ b$	$a\ a$	$b\ b$	$a\ a$					
	7	$b\ b$	$b\ b$	$a\ a$	$a\ a$					
	8	$b\ b$	$b\ b$	$b\ b$	$a\ a$					

I \ D	9.12 $\begin{pmatrix}\varepsilon & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & \varepsilon\end{pmatrix}$				
	No.	φ_{12}	φ_{21}	φ_{31}	φ_{41}
2.1	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$b\ b$	$b\ b$	$b\ b$
	No.	ψ_{12}	ψ_{21}	ψ_{31}	ψ_{41}
2.1′	1	$a\ a$	$a\ a$	$a\ a$	$a\ a$
	2	$a\ a$	$b\ b$	$a\ a$	$a\ a$
	3	$b\ b$	$a\ a$	$a\ a$	$a\ a$
	4	$b\ b$	$b\ b$	$a\ a$	$a\ a$
	5	$a\ a$	$a\ a$	$a\ a$	$b\ b$
	6	$a\ a$	$b\ b$	$a\ a$	$b\ b$
	7	$b\ b$	$a\ a$	$a\ a$	$b\ b$
	8	$b\ b$	$b\ b$	$a\ a$	$b\ b$

Table 9

c-ind. 2 (r-sing.) —$9_{\varepsilon,\alpha}$

I \ D	9.13 $\{\varepsilon,\alpha\}\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$			9.14 $\{\varepsilon,\alpha\}\begin{pmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{pmatrix}$		
	No.	$\varphi_{a\,11}$	$\varphi_{a\,22}$	No.	$\varphi_{a\,12}$	$\varphi_{a\,21}$
2.1	1	a a	a a	1	a a	a a
	2	a a	b b	2	a a	b b

Table 10

c-ind. (r-sing. × l-sing.) — 5

I \ D	5.3 $\begin{pmatrix}\varepsilon & \varepsilon\\ \varepsilon & 0\end{pmatrix}$					5.4 $\begin{pmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{pmatrix}$				
	No.	φ_{11}	φ_{22}	ψ_{11}	ψ_{22}	No.	φ_{12}	φ_{21}	ψ_{12}	ψ_{21}
4.2	1	aacc	aacc	abab	abab	1	aacc	aacc	abab	abab
	2	aacc	aacc	cdcd	abab	2	aacc	aacc	abab	cdcd
	3	bbdd	aacc	cdcd	abab	3	aacc	bbdd	abab	cdcd
6.2	1	bbddff	aaccee	cdcdcd	ababab	1	aaccee	bbddff	ababab	cdcdcd
	2	bbddff	aaccee	ababab	ababab	2	aaccee	bbddff	ababab	ababab
	3	aaccee	aaccee	ababab	ababab	3	aaccee	aaccee	ababab	ababab
	4	aaccee	aaccee	cdcdcd	ababab	4	aaccee	aaccee	ababab	cdcdcd

Table 11

c-ind. 4(r-sing. × l-sing.) — 7

I \ D	7.3 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon\\ \varepsilon & \varepsilon & 0\end{pmatrix}$							7.5 $\begin{pmatrix}0 & \varepsilon & 0\\ \varepsilon & 0 & \varepsilon\end{pmatrix}$						
	No.	φ_{11}	φ_{21}	φ_{32}	ψ_{11}	ψ_{22}	ψ_{32}	No.	φ_{11}	φ_{22}	φ_{31}	ψ_{11}	ψ_{22}	ψ_{31}
4.2	1	aacc	aacc	aacc	abab	abab	abab	1	aacc	aacc	aacc	abab	abab	abab
	2	aacc	aacc	aacc	abab	cdcd	abab	2	aacc	aacc	aacc	abab	abab	cdcd
	3	aacc	aacc	aacc	cdcd	cdcd	abab	3	aacc	aacc	aacc	cdcd	abab	cdcd
	4	bbdd	bbdd	aacc	abab	abab	abab	4	bbdd	aacc	bbdd	abab	abab	abab
	5	bbdd	bbdd	aacc	abab	cdcd	abab	5	bbdd	aacc	bbdd	abab	abab	cdcd
	6	bbdd	bbdd	aacc	cdcd	cdcd	abab	6	bbdd	aacc	bbdd	cdcd	abab	cdcd
	7.4 $\begin{pmatrix}\varepsilon & \varepsilon & \varepsilon\\ \varepsilon & 0 & 0\end{pmatrix}$							7.6 $\begin{pmatrix}\varepsilon & \varepsilon & 0\\ \varepsilon & 0 & \varepsilon\end{pmatrix}$						
	No.	φ_{11}	φ_{22}	φ_{31}	ψ_{11}	ψ_{22}	ψ_{31}	No.	φ_{11}	φ_{21}	φ_{31}	ψ_{11}	ψ_{21}	ψ_{31}
	1	aacc	aacc	aacc	abab	abab	abab	1	aacc	aacc	aacc	abab	abab	abab
	2	aacc	aacc	aacc	abab	abab	cdcd	2	aacc	aacc	aacc	abab	abab	cdcd
	3	aacc	aacc	aacc	cdcd	abab	abab	3	aacc	aacc	aacc	abab	cdcd	abab
	4	aacc	aacc	aacc	cdcd	abab	cdcd	4	aacc	aacc	aacc	abab	cdcd	cdcd
	5	bbdd	aacc	bbdd	abab	abab	abab	5	aacc	bbdd	bbdd	abab	abab	abab
	6	bbdd	aacc	bbdd	abab	abab	cdcd	6	aacc	bbdd	bbdd	abab	abab	cdcd
	7	bbdd	aacc	bbdd	cdcd	abab	abab	7	aacc	bbdd	bbdd	abab	cdcd	abab
	8	bbdd	aacc	bbdd	cdcd	abab	cdcd	8	aacc	bbdd	bbdd	abab	cdcd	cdcd

Table 12

c-ind. (r-sing.) —3_0 —5

I_1	I_2 \ $(S : I_1)$	3_0—5.3			3_0—5.4		
		No.	φ_{11}	φ_{22}	No.	φ_{11}	φ_{22}
2.1	{a b a a}	1	a a	a a	1	a a	a a
	{a b a b}	2	b b	a a	2	a a	b b
3.1	{a b c a a}	1	a a a	a a a	1	a a a	a a a
	{a b c a b}	2	b b b	a a a	2	a a a	b b b
4.1	{a b c d a a}	1	a a a a	a a a a	1	a a a a	a a a a
		2	a b b a	a c a a	2	a c a a	a a b a
		3	a b b a	a c a c	3	a c a a	a a b b
		4	a b b b	a c a a	4	a c a c	a a b a
		5	a b b b	a c a c	5	a c a c	a a b b
	{a b c d a b}	6	a a a a	b b b b	6	a a a a	b b b b
		7	a a a c	b b d d	7	a a a c	b b d b

Table 13

c-ind. 3_0—3_0—5

I_1	(S : I_1) / I_2	5. 3 $\begin{pmatrix}\varepsilon & \varepsilon \\ \varepsilon & 0\end{pmatrix}$					5. 4 $\begin{pmatrix}\varepsilon & 0 \\ 0 & \varepsilon\end{pmatrix}$				
		No.	φ_{11}	φ_{22}	ψ_{11}	ψ_{22}	No.	φ_{12}	φ_{21}	ψ_{12}	ψ_{21}
3	{*aaaaa*}	1	*aaaee*	*aaaad*	*accaa*	*aabaa*	1	*aaaad*	*aaaea*	*aabaa*	*acaaa*
		2	*accee*	*aabad*	*aaaaa*	*aaaaa*	2	*aabad*	*acaea*	*aaaaa*	*aaaaa*
	{*aaabc*}	3	*accee*	*aabad*	*aaaaa*	*aaaaa*	3	*aabad*	*acaea*	*aaaaa*	*aaaaa*

Table 14

c-ind. 4 (**r-sing.** × ***l*-sing.**) —3_0—5

I_1	(S : I_1) / I_2	5. 3 $\begin{pmatrix}\varepsilon & \varepsilon \\ \varepsilon & 0\end{pmatrix}$					5. 4 $\begin{pmatrix}\varepsilon & 0 \\ 0 & \varepsilon\end{pmatrix}$				
		No.	φ_{11}	φ_{22}	ψ_{11}	ψ_{22}	No.	φ_{12}	φ_{21}	ψ_{12}	ψ_{21}
4. 2	φ_e=(*aacc*)	1	*aaccff*	*aaccae*	*ababaa*	*ababaa*	1	*aaccae*	*aaccfa*	*ababaa*	*ababaa*
	φ_f=(*aacc*)	2	*aaccff*	*aaccae*	*ababaa*	*cdcdcc*	2	*aaccae*	*aaccfa*	*ababaa*	*cdcdcc*
	ψ_e=(*abab*)	3	*aaccff*	*aaccae*	*cdcdcc*	*ababaa*	3	*aaccae*	*aaccfa*	*cdcdcc*	*ababaa*
	ψ_f=(*abab*)	4	*aaccff*	*aaccae*	*cdcdcc*	*cdcdcc*	4	*aaccae*	*aaccfa*	*cdcdcc*	*cdcdcc*
	φ_e=(*aacc*)	1	*bbddff*	*aaccae*	*ababaa*	*ababaa*	1	*aaccae*	*bbddfb*	*ababab*	*ababab*
	φ_f=(*bbdd*)	2	*bbddff*	*aaccae*	*ababaa*	*cdcdcc*	2	*aaccae*	*bbddfb*	*ababab*	*cdcdcd*
	ψ_e=(*abab*)	3	*bbddff*	*aaccae*	*cdcdcc*	*ababaa*	3	*aaccae*	*bbddfb*	*cdcdcd*	*ababab*
	ψ_f=(*abab*)	4	*bbddff*	*aaccae*	*cdcdcc*	*cdcdcc*	4	*aaccae*	*bbddfb*	*cdcdcd*	*cdcdcd*

Table 15

c-ind. 2(**sing.**) —3_0—7

(S : I_1) / I_2	(3_0—7. 3) 1				(3_0—7. 3) 2				(3_0—7. 4) 1				(3_0—7. 4) 2			
	No.	φ_{11}	φ_{21}	φ_{32}	No.	φ_{11}	φ_{21}	φ_{32}	No.	φ_{11}	φ_{22}	φ_{31}	No.	φ_{11}	φ_{22}	φ_{31}
{*abaa*}		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*
					2	*b b*	*b b*	*a a*					2	*b b*	*a a*	*b b*
{*abab*}		*b b*	*b b*	*a a*						*b b*	*a a*	*b b*				
		ψ_{11}	ψ_{21}	ψ_{32}		ψ_{11}	ψ_{21}	ψ_{32}		ψ_{11}	ψ_{22}	ψ_{31}		ψ_{11}	ψ_{22}	ψ_{31}
{*abaa*}′	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*
	2	*a a*	*b b*	*a a*					2	*a a*	*a a*	*b b*				
	3	*b b*	*b b*	*a a*					3	*b b*	*a a*	*a a*				
									4	*b b*	*a a*	*b b*				
{*abab*}′						*b b*	*b b*	*a a*						*b b*	*a a*	*b b*

(S : I_1) / I_2	(3_0—7. 5) 1				(3_0—7. 5) 2				(3_0—7. 6) 1				(3_0—7. 6) 2			
	No.	φ_{11}	φ_{22}	φ_{31}	No.	φ_{11}	φ_{22}	φ_{31}	No.	φ_{12}	φ_{21}	φ_{31}	No.	φ_{12}	φ_{21}	φ_{31}
{*abaa*}		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*
					2	*b b*	*a a*	*b b*					2	*b b*	*a a*	*a a*
{*abab*}		*b b*	*a a*	*b b*						*b b*	*a a*	*a a*				
		ψ_{11}	ψ_{22}	ψ_{31}		ψ_{11}	ψ_{22}	ψ_{31}		ψ_{12}	ψ_{21}	ψ_{31}		ψ_{12}	ψ_{21}	ψ_{31}
{*abaa*}′	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*	1	*a a*	*a a*	*a a*		*a a*	*a a*	*a a*
	2	*a a*	*a a*	*b b*					2	*a a*	*b b*	*a a*				
	3	*b b*	*a a*	*b b*					3	*b b*	*a a*	*a a*				
									4	*b b*	*b b*	*a a*				
{*abab*}′						*b b*	*a a*	*b b*						*b b*	*a a*	*a a*

Table 16

c-ind. 2 (r-sing.) —5 (simp.0) —5

(S : I_1) = (5. 3—5. 3)

(S : I_1) / I_2		1		2		3		4		5		6		7		8		9	
	No.	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}
(2. 1—5. 3)1	1 2 3 4	*a a* *a a* *b b* *b b*	*a a* *b b* *a a* *b b*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*
(2. 1—5. 3)2	1 2	*a a* *b b*	*a a* *a a*	*a a*	*a a*	*a a*	*b b*	*a a*	*a a*	*a a*	*b b*	*a a*	*a a*	*b b*	*a a*	*b b*	*b b*	*a a*	*a a*

(S ; I_1) = (5. 3—5. 4)

(S : I_1) / I_2		1		2		3		4		5		6		7	
	No.	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}
(2. 1—5. 3)1	1 2 3	*a a* *a a* *b b*	*a a* *b b* *b b*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*
(2. 1—5. 3)2	1 2 3	*a a* *a a* *b b*	*a a* *b b* *b b*	*a a*	*a a*	*a a*	*b b*	*a a*	*a a*	*a a*	*b b*	*a a*	*a a*	*b b*	*b b*

(S : I_1) = (5. 4—5. 3)

(S : I_1) / I_2	φ_{11}	φ_{22}
(2. 1—5. 4)1	*a a*	*a a*
(2. 1—5. 4)2	*b b*	*b b*

(S : I_1) = (5. 4—5. 4)

(S : I_1) / I_2		1		2		3	
	No.	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}
(2. 1—5. 4)1	1 2 3	*a a* *a a* *b b*	*a a* *b b* *b b*	*a a*	*a a*	*a a*	*a a*
(2. 1—5. 4)2		*a a*	*b b*	*b b*	*b b*	*b b*	*a a*

Table 17

c-ind. 2(r-sing.) —5_0—5

(S : I_1) / I_2	(5_0—5. 3) 1		(5_0—5. 3) 2		(5_0—5. 4)	
	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{12}	φ_{21}
{*abaaaa*}	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*
{*abaabb*}	*b b*	*a a*	*b b*	*a a*	*a a*	*b b*

Table 18

c-ind. 2 (r-sing.) —3_0—3_0—5

(S : I_1) / I_2	(3_0—3_0—5. 3) 1		(3_0—3_0—5. 3) 2		(3_0—3_0—5. 3) 3		(3_0—3_0—5. 4) 1		(3_0—3_0—5. 4) 2		(3_0—3_0—5. 4) 3	
	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{11}	φ_{22}	φ_{12}	φ_{21}	φ_{12}	φ_{21}	φ_{12}	φ_{21}
{*abaaaa*}	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*	*a a*
{*abaaab*}	*b b*	*a a*					*a a*	*b b*				
{*abaaba*}	*a a*	*b b*					*b b*	*a a*				
{*abaabb*}	*b b*	*b b*					*b b*	*b b*				
{*ababab*}			*b b*	*a a*	*b b*	*a a*			*a a*	*b b*	*a a*	*b b*
{*abaacd*}	*d d*	*c c*					*c c*	*d d*				

Table 19

c.d.g. (simp. or g.)—5

I \ D		5.3 $\begin{pmatrix}\varepsilon&\varepsilon\\\varepsilon&0\end{pmatrix}$ No.	φ_{11}	φ_{22}	5.4 $\begin{pmatrix}\varepsilon&0\\0&\varepsilon\end{pmatrix}$ No.	φ_{12}	φ_{21}
group	2.2		a b	a b	1 2	a b b a	a b b a
	3.2		a b c	a b c	1 2	a b c b c a	a b c c a b
	4.4		a b c d	a b c d	1 2	a b c d b c d a	a b c d d a b c
	4.5		a b c d	a b c d	1 2	a b c d b a d c	a b c d b a d c
	5.2		a b c d e	a b c d e	1 2	a b c d e b c d e a	a b c d e e a b c d
	6.5		a b c d e f	a b c d e f	1 2 3	a b c d e f b c d e f a c d e f a b	a b c d e f f a b c d e e f a b c d
	6.6		a b c d e f	a b c d e f	1 2 3	a d c d e f b c a f d e d e f a b c	a b c d e f c a b e f d d e f a b c
non-groups	4.3	1 2	a b a b a b a b	a b a b c d c d	1 2 3 4	a b a b a b a b b a b a b a b a	a b a b c d c d b a b a d c d c
	6.3	1 2	a b a b a b a b a b a b	a b a b a b c d c d c d	1 2 3 4	a b a b a b a b a b a b b d b d b d b d b d b d	a b a b a b c d c d c d b d b d b d d c d c d c
	6.4	1 2	a b c a b c a b c a b c	a b c a b c d e f d e f	1 2 3 4	a b c a b c a b c a b c b c a b c a b c a b c a	a b c a b c d e f d e f c a b c a b f d e f d e

Table 20

c.d.g. (simp. or g.)—7

I \ D		7.3 $\begin{pmatrix}\varepsilon&\varepsilon&\varepsilon\\\varepsilon&\varepsilon&0\end{pmatrix}$ No.	φ_{11}	φ_{21}	φ_{32}	7.4 $\begin{pmatrix}\varepsilon&\varepsilon&\varepsilon\\\varepsilon&0&0\end{pmatrix}$ No.	φ_{11}	φ_{22}	φ_{31}
group	2.2		a b	a b	a b		a b	a b	a b
	3.2		a b c	a b c	a b c		a b c	a b c	a b c
	4.4		a b c d	a b c d	a b c d		a b c d	a b c d	a b c d
	4.5		a b c d	a b c d	a b c d		a b c d	a b c d	a b c d
non-group	4.3	1 2	a b a b a b a b	a b a b a b a b	a d a b c d c d	1 2	a b a b a b a b	a b a b a b a b	a b a b c d c d
	4.3′		a b c d	a b c d	a b c d		a b c d	a b c d	a b c d

I \ D		7.5 $\begin{pmatrix}0&\varepsilon&0\\\varepsilon&0&\varepsilon\end{pmatrix}$ No.	φ_{11}	φ_{22}	φ_{31}	7.6 $\begin{pmatrix}\varepsilon&\varepsilon&0\\\varepsilon&0&\varepsilon\end{pmatrix}$ No.	φ_{12}	φ_{21}	φ_{31}
group	2.2	1 2	a b b a	a b b a	a b b a		a b	a b	a b
	3.2	1 2	a b c b c a	a b c c a b	a b c b c a		a b c	a b c	a b c
	4.4	1 2 3	a b c d c d a b b c d a	a b c d c d a b d a b c	a b c d c d a b b c d a		a b c d	a b c d	a b c d
	4.5	1 2	a b c d b a d c	a b c d b a d c	a b c d b a d c		a b c d	a b c d	a b c d
non-group	4.3	1 2	a b a b a b a b	a b a b c d c d	a b a b a b a b	1 2	a b a b c d c d	a b a b a b a b	a b a b a b a b
	4.3′		a b c d	a b c d	a b c d		a b c d	a b c d	a b c d

Table 21
c. d.g. $2g—9_{\varepsilon}$

I \ D	9.6				9.7				9.8			
	φ_{11}	φ_{21}	φ_{31}	φ_{42}	φ_{11}	φ_{22}	φ_{32}	φ_{42}	φ_{11}	φ_{21}	φ_{32}	φ_{41}
2.2	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*

I \ D	9.9				9.10				9.11				
	φ_{11}	φ_{22}	φ_{32}	φ_{42}	φ_{12}	φ_{21}	φ_{31}	φ_{41}	No.	φ_{12}	φ_{22}	φ_{32}	φ_{41}
2.2	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	*a b*	1 2	*a b* *b a*	*a b* *b a*	*a b* *b a*	*a b* *b a*

I \ D	9.12				
	No.	φ_{12}	φ_{21}	φ_{31}	φ_{41}
2.2	1 2	*a b* *b a*	*a b* *a b*	*a b* *b a*	*a b* *b a*

Table 22 c.d.g. $2g—9_{\varepsilon, \alpha}$

I \ D	9.13			9.14		
	No.	$\varphi_{\alpha 11}$	$\varphi_{\alpha 22}$	No.	$\varphi_{\alpha 12}$	$\varphi_{\alpha 21}$
2.2	1 2	*a b* *b a*	*a b* *b a*	1 2	*a b* *b a*	*a b* *b a*

Table 23 c.d.g. (simp. or g.)—3_0—5

I_1		I_2 \ (S : I_1)	3_0—5.3		3_0—5.4	
			φ_{11}	φ_{22}	φ_{12}	φ_{21}
group	2.2	{*a b a a*}	*a b*	*a b*	*a b*	*a b*
		{*a b a b*}			*b a*	*b a*
		{*a b b b*}	*a b*	*a b*	*a b*	*a b*
	3.2	{*a b c a a*}	*a b c*	*a b c*	*a b c*	*a b c*
		{*a b c a b*}			*c a b*	*b c a*
		{*a b c b b*}	*a b c*	*a b c*	*a b c*	*a b c*
		{*a b c b c*}			*c a b*	*b c a*
	4.4	{*a b c d a a*}	*a b c d*	*a b c d*	*a b c d*	*a b c d*
		{*a b c d b b*}	*a b c d*	*a b c d*	*a b c d*	*a b c d*
		{*a b c d c c*}	*a b c d*	*a b c d*	*a b c d*	*a b c d*
		{*a b c d a b*}			*d a b c*	*b c d a*
		{*a b c d a c*}			*c d a b*	*c d a b*
		{*a b c d b c*}			*d a b c*	*b c d a*
		{*a b c d b d*}			*c d a b*	*c d a b*
	4.5	{*a b c d a a*}	*a b c d*	*a b c d*	*a b c d*	*a b c d*
		{*a b c d b b*}	*a b c d*	*a b c d*	*a b c d*	*a b c d*
		{*a b c d a b*}			*b a d c*	*b a d c*
		{*a b c d c d*}			*b a d c*	*b a d c*
non-group	4.3	{*a b c d a a*}	*a b a b*	*a b a b*	*a b a b*	*a b a b*
		{*a b c d a b*}	*b a b a*	*b a b a*	*b a b a*	*b a b a*
		{*a b c d a c*}	*c d c d*	*a b a b*	*a b a b*	*c d c d*
		{*a b c d a d*}	*d c d c*	*b a b a*	*b a b a*	*d c d c*
		{*a b c d b a*}	*b a b a*	*b a b a*		
		{*a b c d b b*}	*a b a b*	*a b a b*	*a b a b*	*a b a b*
		{*a b c d b c*}	*d c d c*	*b a b a*		
		{*a b c d b d*}	*c d c d*	*a b a b*	*a b a b*	*c d c d*

Table 24

c.d.g. $2g—3_0—7$

I_1	I_2	(S : I_2) / No. of (3_0-7. i) $3 \leqq i \leqq 6$	7. 3 φ_{11}	7. 3 φ_{21}	7. 3 φ_{32}	7. 4 φ_{11}	7. 4 φ_{22}	7. 4 φ_{31}	7. 5 φ_{11}	7. 5 φ_{22}	7. 5 φ_{31}	7. 6 φ_{12}	7. 6 φ_{21}	7. 6 φ_{31}
2. 2	{a b a a}	1	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b
		2 ($i \neq 6$)	a b	a b	a b	a b	a b	a b	a b	a b	a b			
	{a b a b}	1							b a	b a	b a			
		2 ($i \neq 6$)							b a	b a	b a			
	{a b b a}	1							b a	b a	b a			
		2 ($i \neq 6$)							b a	b a	b a			
	{a b b b}	1	a b	a b	a b	a b	a b	a b	a b	a b	a b			
		2 ($i \neq 6$)	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b	a b

Table 25

c.d.g. $2_g—5$ (simp. 0) $—5$

I_1	I_2 (S : I_1)	(5. 3—5. 3) No.	(5. 3—5. 3) φ_{11}	(5. 3—5. 3) φ_{22}	(5. 3—5. 4) No.	(5. 3—5. 4) φ_{12}	(5. 3—5. 4) φ_{21}
2. 2	(2. 2-5. 3)	1	a b	a b	1	a b	a b
		2	a b	a b	2	b a	b a
		3	a b	a b	3	a b	a b
		4	a b	a b	4	a b	a b
		5	a b	a b	5	a b	a b
		6	a b	a b	6	a b	a b
		7	a b	a b	7	a b	a b
		8	a b	a b	8	a b	a b
		9	a b	a b			

I_1	I_2 (S : I_1)	(5. 4—5. 3) φ_{11}	(5. 4—5. 3) φ_{22}	(5. 4—5. 4) 1 No.	(5. 4—5. 4) 1 φ_{12}	(5. 4—5. 4) 1 φ_{21}	(5. 4—5. 4) 2 φ_{12}	(5. 4—5. 4) 2 φ_{21}	(5. 4—5. 4) 3 φ_{12}	(5. 4—5. 4) 3 φ_{21}
2. 2	(2. 2-5. 4) 1	a b	a b	1	a b	a b	a b	a b	a b	a b
				2	b a	b a				
	(2. 2-5. 4) 2	a b	a b		b a	b a	a b	a b	b a	b a

Table 26

c.d.g $2_g—5_0—5$

I_2 (S : I_1)	(5_0—5. 3) 1 φ_{11}	(5_0—5. 3) 1 φ_{22}	(5_0—5. 3) 2 φ_{11}	(5_0—5. 3) 2 φ_{22}	(5_0—5. 4) φ_{12}	(5_0—5. 4) φ_{21}
{a b a a a a}	a b	a b	a b	a b	a b	a b
{a b a b b a}					b a	b a
{a b b b b b}	a b	a b			a b	a b

Table 27

c.d.g. $2_g—3_0—3_0—5$

I_3 (S : I_1)	(3_0—3_0—5. 3) 1 φ_{11}	(3_0—3_0—5. 3) 1 φ_{22}	(3_0—3_0—5. 3) 2 φ_{11}	(3_0—3_0—5. 3) 2 φ_{22}	(3_0—3_0—5. 4) 1 φ_{12}	(3_0—3_0—5. 4) 1 φ_{21}	(3_0—3_0—5. 4) 2 φ_{12}	(3_0—3_0—5. 4) 2 φ_{21}
{a b a a a a}	a b	a b	a b	a b	a b	a b	a b	a b
{a b a a b b}	a b	a b	a b	a b	a b	a b	a b	a b
{a b a b a b}					a b	a b	b a	b a
{a b a b b a}							b a	b a
{a b b b b b}	a b	a b	a b	a b	a b	a b	a b	a b