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ALL SEMIGROUPS OF ORDER AT MOST 5

By

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The main object of this paper is to show a list of all semigroups of order 5, which have been obtained after long computation by hand. However all the discussions as to construction method are remained in another papers. (See references.) For convenience' sake semigroups of order 2, 3, and 4 will be again listed.

In 1951, T. Sakuragi and T. Tamura calculated all semigroups of order 2 and 3 [1] [2], and M. Yamamura computed all semigroups of order 4 for September 1952—January 1953 by elementary method, but T. Akazawa and R. Shibata computed them again in August 1954 by use of a comparatively new method [3]. According to [4], we know that Poole listed some of distinct commutative semigroups and Carman, Harden, and Posey corrected some errors and added some distinct non-commutative semigroups, but their results were incomplete. G.E. Forsythe [4] computed all semigroups of order 4 by electronic method in May and July 1954, and his result is equivalent to ours. Hewitt and Zuckerman also got all semigroups of order 3 and others.

For order 5, unipotent semigroups were gotten by K. Tetsuya and T. Hashimoto in May 1953, the computation of all commutative semigroups were continued by T. Akazawa and R. Shibata for September 1954—January 1955, and all types which include unipotent ones and commutative ones have been obtained by T. Inui and others [1)] a very large calculation for April—July 1955. Independently from us, T.S. Motzkin and J.L. Selfridge computed all semigroups of order 5 with the electronic computer in May 1955, and we received the list which they sent to us three days after we computed them. Our results are completely equivalent to theirs. [2)]

In the previous paper [3] we classified all commutative semigroups by types of greatest semilattice decomposition, and non-commutative semigroups by types of greatest commutativity decomposition, but in the present paper all semigroups are

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2) It took a month to compute to make their tables correspond to ours by hand. We owe correction of our tables to Prof. Motkin and Prof. Selfridge.

classified into categories of types of greatest semilattice decomposition and we use new expression inductively to show multiplication table so that their structures are clarified. In the greatest s-decomposition of a semigroup S, S is decomposed into class sum of s-indecomposable semigroups [9]. S is constructed by types of semilattice, s-indecomposable semigroup, and suitable translations [10] [11]. Further s-indecomposable semigroups are classified into the four categories: c-indecomposable semigroups, unipotent ones with zero, unipotent ones with group, and c-decomposable ones to unipotent. As shown in the list, we have 1160 isomorphically and anti-isomorphically distinct semigroups.

We express our thoughtful thanks to Prof. T.S. Motzkin and Prof. J.L. Selfridge for their kind advice about checking of our list.

References

With respect to each list, see the reference as following.

number of lists		number of references
1		[1]
3		[3]
2, 5		[10] [11]
6 I	c-ind.	[18]
	others	[14] [15] [16] [17]
6 II ~		[7] [9] [10] [11] [12]
7		[6]
8		[3] [16] [17]

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[16] T. Tamura, On finite unipotent semigroup with zero, to appear.

[17] T. Tamura, Note on s-indecomposable semigroups, to appear.

[18] It is proved that c-indecomposable semigroups of order 5 are completely simple. About the structure of completely simple semigroups, see [13].

Errata of the previous paper

"Note on finite semigroups and determination of semigroups of order 4", by T. Tamura, Journal of Gakugei, Tokushima University, Vol. V, 1954, pp. 17-27.

p. 23, line 5 from the bottom, read 188 for 194.

It is for this reason that there are 6 semigroups which are anti-isomorphic to themselves.

I express my thoughtful thanks to Prof. T.S. Motzkin and Prof. J.L. Selfridge for their pointing out this miss.

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Introductory Remarks

General Rules

k_2 .... the k-th semigroup of order 2,

k_n (   ) .... the k-th semigroup of order n ($\geqq 3$), the bracket giving its structure.

k (   ) .... But the suffix n is often omitted if there is no fear of confusion.

k' .... transpose of k.

× .... commutative,

* .... self-dual (anti-isomorphic to itself),

c.i. .... c-indecomposable semigroup,

u.z. .... unipotent semigroup with zero,

u.g. .... unipotent semigroup with group,

c.d.u. .... c-decomposable semigroup to unipotent one.

Semilattices are represented by diagrams, for example,

$a<{b \atop c}$ .... a is a greatest upper bound of b and c, where the ordering $a \geqq b$ means $a=bx$ for some x.

c.i. $k_n(a_1^{(1)} a_2^{(1)} \cdots a_n^{(1)}, a_1^{(2)} a_2^{(2)} \cdots a_n^{(2)}, \cdots, a_1^{(n)} a_2^{(n)} a_n^{(n)}$ ....

$$\begin{vmatrix} a_1^{(1)} & a_2^{(1)} & \cdots\cdots & a_n^{(1)} \\ a_1^{(2)} & a_2^{(2)} & \cdots\cdots & a_n^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ a_1^{(n)} & a_2^{(n)} & \cdots\cdots & a_n^{(n)} \end{vmatrix}$$

u. z. $k_n(a_1^{(1)}\cdots a_{n-2}^{(1)}, a_1^{(2)}\cdots a_{n-2}^{(2)}, \cdots, a_1^{(n-2)}\cdots a_{n-2}^{(n-2)})$ ....

$$\begin{array}{|ccccc|} a & a & \cdots\cdots & & a \\ a & a & & & a \\ \vdots & \vdots & a_1^{(1)} & \cdots\cdots & a_{n-2}^{(1)} \\ \vdots & \vdots & \vdots & & \\ a & a & a_1^{(n-2)} & \cdots & a_{n-2}^{(n-2)} \end{array}$$

u. g. $k_n(j_m, l_{n-m+1}, a_{i_1} a_{i_2} \cdots a_{i_{n-m}})$ a unipotent semigroup whose greatest group is j_m, its difference semigroup modulo j_m is l_{n-m+1}, and a mapping f of elements $a_{m+1}, \cdots, a_n$ not contained in j_m into j_m: $f(a_{m+t}) = a_{m+t} a_1 = a_1 a_{m+t} = a_{i_t}$.

c. d. u. $k_n(j_{n-1}; a_1 \cdots a_{n-1}, b_1 \cdots b_{n-1}; p)$ ....

$$\begin{array}{|c|c|} j_{n-1} & \begin{array}{c} b_1 \\ \vdots \\ b_{n-1} \end{array} \\ \hline a_1 \cdots a_{n-1} & p \end{array}$$

$k_n(i_{n-1})$ ....See examples in remarks of each list.

$k_n(i_{n-1};\ a_{t_1} \cdots a_{t_{n-1}}, a_{s_1} \cdots a_{s_{n-1}})$ ....

$$\begin{array}{c|c|c|} & a_1 \cdots a_{n-1} & a_n \\ \begin{array}{c} a_1 \\ \vdots \\ a_{n-1} \end{array} & i_{n-1} & \begin{array}{c} a_{s_1} \\ \vdots \\ a_{s_{n-1}} \end{array} \\ \hline a_n & a_{t_1} \cdots a_{t_{n-1}} & a_n \end{array}$$

$\left(i_l, j_m;\ \begin{array}{cc} a_1^{(1)} \cdots\cdots a_l^{(1)} & b_1^{(1)} \cdots\cdots b_l^{(1)} \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\ a_1^{(m)} \cdots\cdots a_l^{(m)} & b_1^{(m)} \cdots\cdots b_l^{(m)} \end{array} \right)$ ....

$$\begin{array}{|c|c|} i_l & \begin{array}{ccc} b_1^{(1)} & \cdots\cdots & b_1^{(m)} \\ \vdots & & \vdots \\ b_l^{(1)} & \cdots\cdots & b_l^{(m)} \end{array} \\ \hline \begin{array}{ccc} a_1^{(1)} & \cdots\cdots & a_l^{(1)} \\ \vdots & \cdots\cdots & \\ a_1^{(m)} & \cdots\cdots & a_l^{(m)} \end{array} & j_m \end{array}$$

As far as automorphisms are concerned, we show all ones, if exist, except an identical mapping.

$k_n \quad a_{i_1} a_{i_2} \cdots\cdots a_{i_n}$ ....an automorphism $\begin{pmatrix} a_1 & a_2 & \cdots\cdots a_n \\ a_{i_1} & a_{i_2} & \cdots\cdots a_{i_n} \end{pmatrix}$ of a semigroup k_n where a_t is mapped to a_{i_t},

$k_n \quad a_{i_1} a_{i_2} \cdots\cdots a_{i_n}{}^*$ ....dual-automorphism of k_n.

Translations If a semigroup S is a right (left) unit, any right (left) translation f_z (g_z) of S is given by $f_z(x) = xz$ $(g_z(x) = zx)$. Such translations are omitted in the list.

List 1

a—b ....the diagram of the semilattice of order 2.

Automorphism 1_2 b a......$\begin{pmatrix} a & b \\ b & a \end{pmatrix}$, automorphism of 1_2.

c. i. 1_3(abc abc abc) ....

a	b	c
a	b	c
a	b	c

u. z. 3_3(b) ....

a	a	a
a	a	a
a	a	b

u. g. $5_3(3_2, 2_2$, b)..................

a	b	b
b	a	a
b	a	a

u. g. 6_3(group).....................

a	b	c
b	c	a
c	a	b

c. d. u. $7_3(1_2$; ab aa; a) ......

a	b	a
a	b	a
a	b	a

$8_3(1_2)$ ....semigroup whose greatest s-decomposition is a-(b, c)$_1$; i.e.

a	a	a
a	b	c
a	b	c

$12_3(1_2$; ab bb)......................semigroup whose greatest s-decomposition is (a, b)$_1$-c; i.e.

$$\begin{array}{|ccc|} a & b & b \\ a & b & b \\ a & b & c \end{array}$$

Automorphism

1_3 xyz ....the automorphisms of 1_3 where x, y, z are arbitrary, showing 6 permutations of (a, b, c).

11_3 b a c ....the automorphism of 11_3, $\begin{pmatrix} a & b & c \\ b & a & c \end{pmatrix}$

List 2

$1_2 r.$ aa ab ba bb ....right translations of 1_2 are

$$\begin{pmatrix} a & b \\ a & a \end{pmatrix}, \begin{pmatrix} a & b \\ a & b \end{pmatrix}, \begin{pmatrix} a & b \\ b & a \end{pmatrix}, \begin{pmatrix} a & b \\ b & b \end{pmatrix}.$$

$2_2 r.$ & $l.$ aa ab ....right translations as well as left translations of 2_2;

$$\begin{pmatrix} a & b \\ a & a \end{pmatrix}, \begin{pmatrix} a & b \\ a & b \end{pmatrix}.$$

$7_3\ l.$ aba abb abc ....left translations of 7_3 are

$$\begin{pmatrix} a & b & c \\ a & b & a \end{pmatrix}, \begin{pmatrix} a & b & c \\ a & b & b \end{pmatrix}, \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}.$$

$1_3 r.$ xyz.............................arbitrary mappings of (abc) into itself are right translations of 1_3.

List 3

20 (cyclic group).................. $\begin{array}{|cccc|} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{array}$

21 ($3_2 \times 3_2$ group) .... $\begin{array}{|cccc|} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{array}$

24 ($1_2 \times 3_2$).......................... $\begin{array}{|cccc|} a & b & c & d \\ a & b & c & d \\ c & d & a & b \\ c & d & a & b \end{array}$

a-(b, c, d)$_i$ 28 (1) .... $\begin{array}{|cccc|} a & a & a & a \\ a & b & c & d \\ a & b & c & d \\ a & b & c & d \end{array}$

$a <^{(b,\ c)_i}_{d}$ 91 (1) .... $\begin{array}{|cccc|} a & a & a & a \\ a & b & c & a \\ a & b & c & a \\ a & a & a & a \end{array}$

$\overbrace{(a,\ b)<^{(c)}_{d}}^{i}$ $k(i;\ a_1 a_2 a_3,\ b_1 b_2 b_3)$ ...... $\begin{array}{|c|c|} & b_1 \\ i_3 & b_2 \\ & b_3 \\ \hline a_1\ a_2\ a_3 & d \end{array}$ where i_3 is the form (a, b)-c.

a-b-(c, d)$_i$ $k(i)$ .... $\begin{array}{|cc|cc|} a & a & a & a \\ a & b & b & b \\ \cline{3-4} a & b & \multicolumn{2}{c|}{} \\ a & b & \multicolumn{2}{c|}{i} \end{array}$

a-$\overbrace{(b,\ c)\text{-}(d)}^{i}$ $k(i)$ .... $\begin{array}{|c|ccc|} a & a & a & a \\ \cline{2-4} a & & & \\ a & & i & \\ a & & & \end{array}$ where i is the form (b, c)-(d).

$\overbrace{(a,\ b)\text{-}(c)}^{i}$-d $k(i\!:\ a_1 a_2 a_3,\ b_1 b_2 b_3)$......... $\begin{array}{|c|c|} & b_1 \\ i & b_2 \\ & b_3 \\ \hline a_1\ a_2\ a_3 & d \end{array}$

The final table in the List 3 shows the numbers written in the [3] for the numbers in the present list, for example, No. 1 here is No. 20 in the former.

List 4

1 xyzu ....every mapping of (abcd) into (abcd) is an automorphism of 1_4.

2 badc 2 cdab 2 dcba 2 acbd*......all automorphisms of 2_4 are $\begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix}$ $\begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix}$

$\begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix}$ and dual automorphism of 2_4 is $\begin{pmatrix} a & b & c & d \\ a & c & b & d \end{pmatrix}$

3 axyz ....x, y, z vary through a, b, c, d.

List 5

Since it is complicated to list all translations, we show only the contractions of translations to a one-sided base.

$2r.$ $\begin{pmatrix} c & d \\ \frown & \frown \\ c\ d & c\ d \end{pmatrix}$ ....contractions of right translations to a right base;
$\begin{pmatrix} c & d \\ c & c \end{pmatrix}$, $\begin{pmatrix} c & d \\ c & d \end{pmatrix}$, $\begin{pmatrix} c & d \\ d & c \end{pmatrix}$, $\begin{pmatrix} c & d \\ d & d \end{pmatrix}$.

$2l.$ $\begin{pmatrix} b & d \\ \frown & \frown \\ b\ d & b\ d \end{pmatrix}$ ....contractions of left translations to a left base;
$\begin{pmatrix} b & d \\ b & b \end{pmatrix}$, $\begin{pmatrix} b & d \\ b & d \end{pmatrix}$, $\begin{pmatrix} b & d \\ d & b \end{pmatrix}$, $\begin{pmatrix} b & d \\ d & d \end{pmatrix}$.

$1r.$ $\begin{pmatrix} a & b & c & d \\ x & y & z & u \end{pmatrix}$......................x, y, z, u are arbitrary.

3 $\begin{pmatrix} b & c & d \\ x & y & z \end{pmatrix}$ ....there is no distinction between right and left because 3_4 is commutative.

List 6

135 (cyclic group) ....

a	b	c	d	e
b	c	d	e	a
c	d	e	a	b
d	e	a	b	c
e	a	b	c	d

$a <\begin{matrix} (b,\ c)_i \\ (d,\ e)_j \end{matrix}$ 596 (1, 2)......

a	a	a	a	a
a	i		a	a
a			a	a
a	a	a	j	
a	a	a		

$(a, b)_i$-c-$(d, e)_j$ $k\begin{pmatrix} & a_1 a_2 & a_1' a_2' \\ i, j; & b_1 b_2 & b_1' b_2' \\ & c_1 c_2 & c_1' c_2' \end{pmatrix}$.........

i		a_1'	b_1'	c_1'
		a_2'	b_2'	c_2'
a_1	a_2	c	c	c
b_1	b_2	c	j	
c_1	c_2	c		

$\overbrace{(a, b)}^{i}$$<\begin{matrix}(c)\\(d)\\e\end{matrix}$ $k(i;\ a_1 a_2 a_3 a_4,\ b_1 b_2 b_3 b_4)$ ......

i				b_1
				b_2
				b_3
				b_4
a_1	a_2	a_3	a_4	e

where i is the form

$(a, b) <\begin{matrix}(c)\\(d)\end{matrix}$

$a <\overbrace{\begin{matrix}(b,\ c)\\ e\end{matrix}}^{i}> (d)$ $k(i)$ ....

a	a	a	a	a
a	i			a
a				a
a				e
a	a	a	e	e

where i is the form

(b, c)-(d)

$\overbrace{(a, b) <\begin{matrix}(c)\\(d)\end{matrix}}^{i}> e$ $k(i;\ a_1 a_2 a_3 a_4,\ b_1 b_2 b_3 b_4)$ ......i is the form $(a, b) <\begin{matrix}(c)\\(d)\end{matrix}$

List 7

Our number 1 is equivalent to Motzkin's number 994, and our 12 to his 17. Here "equivalent" means to "isomorphic or anti-isomorphic"

List 8

We show l-ordering and r-ordering of u.z. of order 5, and greatest c-decomposition of c.d.u., for example, 136∼139 are homomorphic to a commutative 2 and this c-decomposition is greatest.

List 1 Semigroups of Order 2, 3

Order 2

c.i.	
a	b
a	b

1_2

u.z.	
a	a
a	a

$2_2^{\times}$

u.g.	
a	b
b	a

$3_2^{\times}$

a–b	
a	a
a	b

$4_2^{\times}$

Automorphism 1_2 ba

Order 3

I s-indecomposable

c.i. 1_3(abc abc abc) u.z. $2_3^{\times}$(a) u.z. $3_3^{\times}$(b) u.g. $4_3^{\times}$(3_2, 2_2, a) u.g. $5_3^{\times}$(3_2, 2_2, b) u.g. $6_3^{\times}$(group) c.d.u. 7_3(1_2; ab aa; a)

II a–$(b, c)_i$

8_3(1_2) $9_3^{\times}$(2_2) $10_3^{\times}$(3_2)

III $(a, b)_i$–c

11_3(1_2; ab ab) 12_3(1_2; ab bb) $13_3^{\times}$(2_2; aa aa) 14_3(2_2; aa ab) $15_3^{\times}$(2_2; ab ab) $16_3^{\times}$(3_2; ab ab)

IV Semilattices

$17_3^{\times}$ a $<$ b, c $18_3^{\times}$ a—b—c

Automorphisms

1_3 x y z 2_3 a c b 6_3 a c b 8_3 a c b 11_3 b a c 17_3 a c b

List 2 Translations of Semigroups of Order 2, 3

1_2 *r.* aa ab ba bb, 2_2 *r.* & *l.* aa ab 3_2 *r.* & *l.* ab ba 4_2 *r.* & *l.* aa ab 1_3 *r.* xyz 2_3 *r.* & *l.* aaa aab aac aba abb abc aca acb acc 3_3 *r.* & *l.* aaa aab abc 4_3 *r.* & *l.* aba bab abc 5_3 *r.* & *l.* abb baa abc 7_3 *r.* aaa bab aac aba bbb abc 7_3 *l.* aba abb abc 8_3 *r.* aaa aab aac aba abb abc aca acb acc 9_3 *r.* & *l.* aaa abb abc 11_3 *r.* aaa abb abc baa bbb bbc 13_3 *r.* & *l.* aaa aac aba abc 14_3 *l.* aaa aab aac aba abb abc aca acb acc 17_3 *r.* & *l.* aaa aac aba abc

List 3 Semigroups of Order 4

I s-indecomposable

c-ind.

1 (abcd abcd abcd abcd) 2* (abab abab cdcd cdcd)

u.z.

$3^{\times}$(aa aa) $4^{\times}$(aa ab) 5*(aa ba) 6(aa bb) $7^{\times}$(ab ba) $8^{\times}$(ab bb) $9^{\times}$(ba ab) 10*(ba bb) $11^{\times}$(bb bb) $12^{\times}$(ab bc)

u.g.

$13^{\times}$(3_2, 2_3, aa) $14^{\times}$(3_2, 2_3, ab) $15^{\times}$(3_2, 2_3, bb) $16^{\times}$(3_2, 3_3, aa) $17^{\times}$(3_2, 3_3, ab) $18^{\times}$(6_3, 2_2, a) $19^{\times}$(6_3, 2_2, b) $20^{\times}$(cyclic group) $21^{\times}$($3_2 \times 3_2$ group)

c.d.u.

22(1_3; abc aaa; a) 23(1_3; abc aab; a) 24($1_2 \times 3_2$) 25(7_3; aba aaa; a) 26(7_3; aba aaa; c) 27(7_3; aba bbb; b)

II a–$(b, c, d)_i$

28(1) $29^{\times}$(2) $30^{\times}$(3) $31^{\times}$(4) $32^{\times}$(5) $33^{\times}$(6) 34(7)

III $(\mathbf{a}, \mathbf{b})_i$–$(\mathbf{c}, \mathbf{d})_j$

35$\left(1,1;\ \begin{matrix}ab & aa\\ ab & aa\end{matrix}\right)$	36$\left(1,1;\ \begin{matrix}ab & aa\\ ab & bb\end{matrix}\right)$	37$\left(1,1;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	38$\left(1,1';\ \begin{matrix}ab & aa\\ ab & aa\end{matrix}\right)$	39$\left(1,1';\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$
40$\left(1,2;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	41$\left(1,2;\ \begin{matrix}ab & bb\\ ab & bb\end{matrix}\right)$	42$\left(1,3;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	43$\left(1,3;\ \begin{matrix}ab & ab\\ ab & ba\end{matrix}\right)$	44$\left(1,3;\ \begin{matrix}ab & bb\\ ab & bb\end{matrix}\right)$
45$\left(2,1;\ \begin{matrix}aa & aa\\ aa & aa\end{matrix}\right)$	46$\left(2,1;\ \begin{matrix}ab & aa\\ ab & aa\end{matrix}\right)$	47$\left(2,1;\ \begin{matrix}aa & ab\\ aa & ab\end{matrix}\right)$	48$\left(2,1;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	49$^\times\left(2,2;\ \begin{matrix}aa & aa\\ aa & aa\end{matrix}\right)$
50$\left(2,2;\ \begin{matrix}aa & ab\\ aa & ab\end{matrix}\right)$	51$^\times\left(2,2;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	52$^\times\left(2,3;\ \begin{matrix}aa & aa\\ aa & aa\end{matrix}\right)$	53$\left(2.3;\ \begin{matrix}ab & aa\\ ab & aa\end{matrix}\right)$	54$^\times\left(2,3;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$
55$\left(3,1;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	56$^\times\left(3,2;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	57$^\times\left(3,3;\ \begin{matrix}ab & ab\\ ab & ab\end{matrix}\right)$	58$^\times\left(3,3;\ \begin{matrix}ab & ab\\ ba & ba\end{matrix}\right)$	

IV $(\mathbf{a}, \mathbf{b}, \mathbf{c})_i$–$\mathbf{d}$

59(1; abc aaa)	60(1; abc aba)	61(1; abc abc)	62$^\times$(2; aaa aaa)	63(2; aaa aba)
64(2; aaa abb)	65(2; aaa abc)	66$^\times$(2; aac aac)	67*(2; aac aba)	68(2; aac abc)
69$^\times$(2; abb abb)	70(2; abc acc)	71$^\times$(2; abc abc)	72$^\times$(3; aaa aaa)	73$^\times$(3; abc abc)
74$^\times$(4; aba aba)	75(4; aba abc)	76$^\times$(4; abc abc)	77$^\times$(5; abb abb)	78(5; abb abc)
79$^\times$(5; abc abc)	80$^\times$(6; abc abc)	81(7; aba aaa)	82(7; aba aac)	83(7; aba aba)
84(7; aba abc)	85(7; aba bbb)	86(7; abc aaa)	87(7; abc aac)	88(7; abc aba)
89(7; abc abc)	90(7; abc bbb)			

V $\mathbf{a}<\begin{smallmatrix}(\mathbf{b},\ \mathbf{c})_i\\ \mathbf{d}\end{smallmatrix}$

91(1) 92$^\times$(2) 93$^\times$(3)

VI $\overbrace{(\mathbf{a}, \mathbf{b})<\begin{smallmatrix}(\mathbf{c})\\ \mathbf{d}\end{smallmatrix}}^{i}$

94(11; aba aaa)	95(12; abb aaa)	96(12; abb bbb)	97$^\times$(13; aaa aaa)	98(14; aaa aaa)
99(14; aba aaa)	100(14; abb aaa)	101$^\times$(15; aaa aaa)	102$^\times$(16; aba aba)	

VII $\mathbf{a}$–$\mathbf{b}$–$(\mathbf{c}, \mathbf{d})_i$

103(1) 104$^\times$(2) 105$^\times$(3)

VIII $\mathbf{a}\text{–}\overbrace{(\mathbf{b}, \mathbf{c})\text{–}(\mathbf{d})}^{i}$

106(11) 107(12) 108$^\times$(13) 109(14) 110$^\times$(15) 111$^\times$(16)

IX $\overbrace{(\mathbf{a}, \mathbf{b})\text{–}(\mathbf{c})}^{i}\text{–}\mathbf{d}$

112(11; abc abc)	113(12; abc abc)	114(12; abc bbc)	115$^\times$(13; aac aac)	116(13; aac abc)
117$^\times$(13; abc abc)	118(14; aac abc)	119(14; abc abc)	120$^\times$(15; abc abc)	121$^\times$(16; abc abc)

X Semilattice

122$^\times$ $a<\begin{smallmatrix}b\\ c\\ d\end{smallmatrix}$ 123$^\times$ $a<\begin{smallmatrix}b\\ c\end{smallmatrix}>d$ 124$^\times$ $a<\begin{smallmatrix}b\text{—}d\\ c\end{smallmatrix}$ 125$^\times$ $a\text{—}b<\begin{smallmatrix}c\\ d\end{smallmatrix}$ 126$^\times$ a—b—c—d

Former Numbers for New Numbers

	0	1	2	3	4	5	6	7	8	9
000		020	021	001	003	010	004	009	006	008
010	007	005	002	013	014	011	012	015	016	017
020	018	019	036	037	029	044	043	045	035	125
030	126	124	123	122	084	022	024	023	027	028
040	082	081	046	047	048	080	025	026	052	110
050	083	109	108	049	107	056	106	104	105	032
060	033	034	118	078	079	030	119	031	053	120
070	054	121	117	116	115	055	114	112	057	113
080	111	075	038	077	039	076	041	051	040	050
090	042	074	103	102	068	070	069	101	071	072
100	073	100	099	067	097	098	064	065	096	066
110	095	094	058	059	060	093	063	092	061	062
120	091	090	089	088	087	086	085			

List 4 Automorphisms of Semigroups of Order 4

1 xyzu	2 badc	2 cdab	2 dcba	2 acbd*	3 axyz	5 abdc*	7 abdc
9 abdc	10 abdc*	11 abdc	13 abdc	15 abdc	18 acbd	20 adcb	21 axyz
22 acbd	24 badc	25 abdc	27 badc	28 axyz	29 abdc	33 abdc	35 abdc
36 badc	37 abdc	37 badc	37 bacd	38 abdc	39 bacd	39 abdc	39 badc
40 bacd	42 bacd	43 bacd	45 abdc	46 abdc	47 abdc	48 abdc*	55 abdc
59 acbd	61 xyzd	62 acbd	65 acbd	67 acbd*	71 acbd	80 acbd	91 acbd
95 abdc	96 badc	97 abdc	99 abdc*	100 abdc*	102 abdc	103 abdc	106 acbd
112 bacd	122 axyz	123 acbd	125 abdc				

List 5 Translations of Semigroup of Order 4

$$1r\begin{pmatrix} abcd \\ xyzu \end{pmatrix} \quad 2r\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix} \quad 2l\begin{pmatrix} b & d \\ \overgroup{bd} & \overgroup{bd} \end{pmatrix} \quad 3\begin{pmatrix} bcd \\ xyz \end{pmatrix} \quad 4\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 5r\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abd} \end{pmatrix} \quad 5l\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix}$$

$$6r\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix} \quad 6l\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 7\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ c & d \end{pmatrix} \quad 8\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ c & d \end{pmatrix} \quad 9\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ c & d \end{pmatrix}$$

$$10r\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ c & d \end{pmatrix} \quad 10l\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ c & d \end{pmatrix} \quad 11\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix} \quad 12\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 13\begin{pmatrix} c & d \\ \overgroup{acd} & \overgroup{acd} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix}$$

$$14\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{bd} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{bd} & \overgroup{ac} \end{pmatrix} \quad 15\begin{pmatrix} c & d \\ \overgroup{bcd} & \overgroup{bcd} \end{pmatrix}\begin{pmatrix} c & d \\ a & a \end{pmatrix} \quad 16\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 17\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 18\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 19\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix}$$

$$22r\begin{pmatrix} b & c & d \\ \overgroup{abc} & \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 22l\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 23r\begin{pmatrix} b & c & d \\ b & \overgroup{abc} & \overgroup{bd} \end{pmatrix}\begin{pmatrix} b & c & d \\ a & \overgroup{abc} & a \end{pmatrix}\begin{pmatrix} b & c & d \\ c & \overgroup{abc} & c \end{pmatrix} \quad 23l\begin{pmatrix} d \\ \overgroup{abd} \end{pmatrix} \quad 24r\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix}$$

$$25r\begin{pmatrix} b & c & d \\ \overgroup{ab} & b & b \end{pmatrix}\begin{pmatrix} b & c & d \\ \overgroup{ab} & \overgroup{acd} & \overgroup{acd} \end{pmatrix} \quad 25l\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 26r\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abcd} \end{pmatrix} \quad 26l\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 27r\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix}$$

$$27l\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 28r\begin{pmatrix} b & c & d \\ \overgroup{abcd} & \overgroup{abcd} & d \end{pmatrix} \quad 29\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{bcd} & \overgroup{bcd} \end{pmatrix} \quad 30\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 31\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 32\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix}$$

$$34r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 34l\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 35r\begin{pmatrix} b & c & d \\ \overgroup{ab} & \overgroup{acd} & \overgroup{acd} \end{pmatrix}\begin{pmatrix} b & c & d \\ \overgroup{ab} & b & b \end{pmatrix} \quad 36r\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 37r\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix}$$

$$38r\begin{pmatrix} b & c \\ \overgroup{ab} & \overgroup{abc} \end{pmatrix} \quad 38l\begin{pmatrix} c & d \\ \overgroup{acd} & \overgroup{acd} \end{pmatrix} \quad 39l\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 40r\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 40l\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 41r\begin{pmatrix} a & d \\ \overgroup{ab} & \overgroup{abcd} \end{pmatrix} \quad 41l\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix}$$

$$44r\begin{pmatrix} a & c \\ a & \overgroup{abcd} \end{pmatrix}\begin{pmatrix} a & c \\ b & \overgroup{bcd} \end{pmatrix} \quad 45r\begin{pmatrix} b & c & d \\ \overgroup{ab} & \overgroup{acd} & \overgroup{acd} \end{pmatrix} \quad 45l\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{ad} \end{pmatrix} \quad 46r\begin{pmatrix} b & c & d \\ \overgroup{abcd} & \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 47r\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix}$$

$$47l\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abd} \end{pmatrix} \quad 48r\begin{pmatrix} c & d \\ \overgroup{ab} & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{cd} & \overgroup{cd} \end{pmatrix} \quad 49\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abcd} \end{pmatrix} \quad 50r\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 50l\begin{pmatrix} b & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 51\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix}$$

$$52\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{acd} \end{pmatrix} \quad 53r\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abcd} \end{pmatrix} \quad 55r\begin{pmatrix} c & d \\ \overgroup{acd} & \overgroup{acd} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 56\begin{pmatrix} d \\ \overgroup{abcd} \end{pmatrix} \quad 59r\begin{pmatrix} b & c & d \\ \overgroup{abc} & \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 60r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix}$$

$$62\begin{pmatrix} b & c & d \\ \overgroup{abc} & \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 63r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 63l\begin{pmatrix} b & c & d \\ \overgroup{ad} & \overgroup{ac} & \overgroup{abd} \end{pmatrix} \quad 64r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 64l\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abd} \end{pmatrix} \quad 65l\begin{pmatrix} bcd \\ xyz \end{pmatrix}$$

$$66\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{acd} \end{pmatrix} \quad 67r\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{acd} \end{pmatrix} \quad 67l\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abd} \end{pmatrix} \quad 68l\begin{pmatrix} b & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix} \quad 69\begin{pmatrix} c & d \\ a & \overgroup{ab} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{bc} & d \end{pmatrix} \quad 70r\begin{pmatrix} b & d \\ \overgroup{abc} & \overgroup{abcd} \end{pmatrix}$$

$$72\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 74\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{ad} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 75l\begin{pmatrix} c & d \\ \overgroup{acd} & \overgroup{acd} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 77\begin{pmatrix} c & d \\ a & b \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{bc} & \overgroup{ad} \end{pmatrix} \quad 78l\begin{pmatrix} c & d \\ \overgroup{ad} & \overgroup{bc} \end{pmatrix}\begin{pmatrix} c & d \\ \overgroup{bc} & \overgroup{ad} \end{pmatrix}$$

$$81r\begin{pmatrix} b & c & d \\ \overgroup{ab} & \overgroup{ac} & \overgroup{ad} \end{pmatrix}\begin{pmatrix} b & c & d \\ \overgroup{ab} & b & b \end{pmatrix} \quad 81l\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 82r\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abd} \end{pmatrix} \quad 82l\begin{pmatrix} c & d \\ \overgroup{acd} & \overgroup{acd} \end{pmatrix} \quad 83r\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{ad} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 83l\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{abd} \end{pmatrix}$$

$$84l\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 85r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abd} \end{pmatrix} \quad 85l\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{bd} \end{pmatrix} \quad 86r\begin{pmatrix} b & c & d \\ \overgroup{ab} & \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 87r\begin{pmatrix} b & d \\ \overgroup{ab} & \overgroup{abcd} \end{pmatrix}$$

$$88r\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{acd} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 90r\begin{pmatrix} c & d \\ \overgroup{abcd} & \overgroup{abcd} \end{pmatrix} \quad 91r\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{ad} \end{pmatrix} \quad 91l\begin{pmatrix} b & c & d \\ \overgroup{abc} & \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 92\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 93\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix}$$

$$94r\begin{pmatrix} c & d \\ \overgroup{ac} & \overgroup{ad} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix} \quad 94l\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{ad} \end{pmatrix} \quad 95r\begin{pmatrix} a & c & d \\ \overgroup{ab} & a & a \end{pmatrix}\begin{pmatrix} a & c & d \\ \overgroup{ab} & \overgroup{bc} & \overgroup{bd} \end{pmatrix} \quad 95l\begin{pmatrix} c & d \\ \overgroup{bc} & \overgroup{bd} \end{pmatrix} \quad 96r\begin{pmatrix} c & d \\ \overgroup{abc} & \overgroup{abd} \end{pmatrix} \quad 96l\begin{pmatrix} c & d \\ \overgroup{bc} & \overgroup{ad} \end{pmatrix}$$

$97\begin{pmatrix} b & c & d \\ \overset{\frown}{ab} & \overset{\frown}{ac} & \overset{\frown}{ad} \end{pmatrix}$ $98r\begin{pmatrix} c & d \\ \overset{\frown}{ac} & \overset{\frown}{ad} \end{pmatrix}$ $98l\begin{pmatrix} b & c & d \\ \overset{\frown}{abc} & \overset{\frown}{abc} & \overset{\frown}{ad} \end{pmatrix}$ $99r\begin{pmatrix} c & d \\ \overset{\frown}{ac} & \overset{\frown}{abd} \end{pmatrix}$ $99l\begin{pmatrix} c & d \\ \overset{\frown}{abc} & \overset{\frown}{ad} \end{pmatrix}$ $100r\begin{pmatrix} c & d \\ \overset{\frown}{ac} & \overset{\frown}{abd} \end{pmatrix}$

$100l\begin{pmatrix} c & d \\ \overset{\frown}{abc} & \overset{\frown}{ad} \end{pmatrix}$ $101\begin{pmatrix} c & d \\ \overset{\frown}{abc} & \overset{\frown}{ad} \end{pmatrix}$ $102\begin{pmatrix} c & d \\ \overset{\frown}{ac} & \overset{\frown}{ad} \end{pmatrix}\begin{pmatrix} c & d \\ b & b \end{pmatrix}$ $103r\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overset{\frown}{bcd} & \overset{\frown}{bcd} \end{pmatrix}$ $104\begin{pmatrix} d \\ \overset{\frown}{abcd} \end{pmatrix}$ $107r\begin{pmatrix} b & d \\ \overset{\frown}{abc} & \overset{\frown}{abcd} \end{pmatrix}$

$108\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overset{\frown}{bc} & \overset{\frown}{bd} \end{pmatrix}$ $109l\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overset{\frown}{bcd} & \overset{\frown}{bcd} \end{pmatrix}$ $113r\begin{pmatrix} a & d \\ \overset{\frown}{ab} & \overset{\frown}{abcd} \end{pmatrix}$ $115\begin{pmatrix} b & d \\ \overset{\frown}{ab} & \overset{\frown}{acd} \end{pmatrix}$ $116l\begin{pmatrix} b & d \\ \overset{\frown}{ab} & \overset{\frown}{abcd} \end{pmatrix}$

$118l\begin{pmatrix} b & d \\ \overset{\frown}{abc} & \overset{\frown}{abcd} \end{pmatrix}$ $122\begin{pmatrix} b & c & d \\ \overset{\frown}{ab} & \overset{\frown}{ac} & \overset{\frown}{ad} \end{pmatrix}$ $124\begin{pmatrix} c & d \\ \overset{\frown}{ac} & \overset{\frown}{abd} \end{pmatrix}$ $125\begin{pmatrix} c & d \\ a & a \end{pmatrix}\begin{pmatrix} c & d \\ \overset{\frown}{bc} & \overset{\frown}{bd} \end{pmatrix}$

List 6 Semigroups of Order 5

I s-indecomposable

c-ind.

1(abcde abcde abcde abcde abcde) 2*(aaaaa aaacb abcaa acdaa aaade)
3*(aaaaa abcaa abccb aeddc acdaa)

u. z.

4$^\times$(aaa aaa aaa)	5(aaa aaa abc)	6$^\times$(aaa aba aac)	7*(aaa aab aca)	8(aaa aba acc)
9(aaa aab acc)	10(aaa abb acc)	11$^\times$(aaa abc acc)	12$^\times$(aaa abc aca)	13*(aaa acb acc)
14*(aaa acb aac)	15$^\times$(aaa abc acb)	16$^\times$(aaa aab abd)	17$^\times$(aaa aaa aab)	18*(aaa aaa aba)
19(aaa aaa abb)	20(aaa aaa bba)	21(aaa aaa bbb)	22(aab aaa bbc)	23$^\times$(aab aab bbc)
24$^\times$(aab aba bac)	25(aab aba bbc)	26$^\times$(aab abb bbc)	27$^\times$(abb bcc bcc)	28$^\times$(aaa aab aba)
29$^\times$(aaa aab abb)	30$^\times$(aaa aba aab)	31*(aaa aba abb)	32$^\times$(aaa abb abb)	33*(aaa aab baa)
34*(aaa bab baa)	35*(aaa aab bab)	36*(aaa bab bab)	37*(aaa baa aab)	38(aaa baa bab)
39(aaa aab bba)	40(aaa bab bba)	41(aaa abb bba)	42(aaa bbb aba)	43(aaa bbb bba)
44(aaa abb baa)	45(aaa abb bbb)	46(aaa baa bbb)	47(aaa bba aab)	48(aaa bba bab)
49(aaa bba abb)	50(aaa bbb aab)	51(aaa bba bbb)	52(aaa bbb bbb)	53*(aba aab baa)
54*(aba aab bba)	55$^\times$(aba bab aba)	56*(aba bab bba)	57$^\times$(abb bab bba)	58$^\times$(baa aab aba)
59(baa aab bba)	60(baa bab bba)	61$^\times$(bab aab bba)	62*(bab baa aba)	63*(bab bab aba)
64(bab baa bba)	65(bab bab bba)	66$^\times$(bbb baa baa)	67*(bbb baa bba)	68$^\times$(bbb bab bba)
69$^\times$(baa abb aba)	70*(baa abb baa)	71(baa abb bba)	72(baa bbb aba)	73*(baa bbb baa)
74(baa bbb bba)	75$^\times$(bab abb bba)	76*(bab bba aba)	77(bab bbb aba)	78*(bab bbb bba)
79$^\times$(bba bbb aba)	80*(bba bbb baa)	81(bba bbb bba)	82$^\times$(bbb bbb bba)	83$^\times$(baa aba aab)
84*(baa aba abb)	85$^\times$(baa abb abb)	86(baa aba bbb)	87*(baa abb bab)	88(baa abb bbb)
89*(baa bba bbb)	90(baa bbb bbb)	91$^\times$(bab abb bbb)	92*(bab bba abb)	93*(bab bbb bbb)
94*(bab bbb bbb)	95$^\times$(bbb bbb bbb)	96$^\times$(aab abc bcd)		

u. g.

97$^\times$(3_2, 3_4, aaa)	98$^\times$(3_2, 3_4, aab)	99$^\times$(3_2, 3_4, abb)	100$^\times$(3_2, 3_4, bbb)	101$^\times$(3_2, 4_4, aaa)
102$^\times$(3_2, 4_4, aba)	103$^\times$(3_2, 4_4, aab)	104$^\times$(3_2, 4_4, abb)	105*(3_2, 5_4, aaa)	106*(3_2, 5_4, abb)
107(3_2, 5_4, bab)	108(3_2, 6_4, aaa)	109(3_2, 6_4, abb)	110$^\times$(3_2, 7_4, aaa)	111$^\times$(3_2, 7_4, bab)
112$^\times$(3_2, 7_4, abb)	113$^\times$(3_2, 8_4, aaa)	114$^\times$(3_2, 8_4, abb)	115$^\times$(3_2, 9_4, aaa)	116$^\times$(3_2, 9_4, aab)
117$^\times$(3_2, 9_4, abb)	118*(3_2, 10_4, aaa)	119*(3_2, 10_4, abb)	120$^\times$(3_2, 11_4, aaa)	121$^\times$(3_2, 11_4, abb)
122$^\times$(3_2, 12_4, aaa)	123$^\times$(3_2, 12_4, bab)	124$^\times$(6_3, 2_3, aa)	125$^\times$(6_3, 2_3, ab)	126$^\times$(6_3, 2_3, bb)
127$^\times$(6_3, 2_3, bc)	128$^\times$(6_3, 3_3, aa)	129$^\times$(6_3, 3_3, bc)	130$^\times$(20_4, 2_2, a)	131$^\times$(20_4, 2_2, b)
132$^\times$(20_4, 2_2, c)	133$^\times$(21_4, 2_2, a)	134$^\times$(21_4, 2_2, b)	135$^\times$(cyclic group)	

c. d. u.

136(1_4, abcd aaaa; a)	137(1_4, abcd aaba; a)	138(1_4, abcd aabb; a)	139*(2_4, abab aacc; a)
140(24_4, abcd aacc; a)	141(24_4, cdab ccaa; a)	142(22_4, abca aaaa; a)	143(22_4, abca aaba; a)
144(22_4, abca bbab; b)	145(22_4, abca bbbb; b)	146(23_4, abca aaba; a)	147(23_4, abca bbab; b)
148(23_4, abcb cccc; c)	149(25_4, abaa aaac; a)	150(25_4, abac bbbb; b)	151(25_4, abac aaaa; c)
152(25_4, abaa aaac; c)	153(26_4, abac bbbb; b)	154(26_4, abaa aaac; c)	155(22_4, abca aaaa; d)
156(22_4, abca aaba; d)	157(25_4, abaa aaaa; a)	158(25_4, abaa bbbb; b)	159(25_4, abaa aaaa; c)
160(27_4, abab aaaa; c)	161(25_4, abac aaac; a)	162(26_4, abac aaac; a)	163(26_4, abaaa aaa; c)
164(26_4, abac aaac; c)	165(25_4, abac aaac; d)		

II a-(b, c, d, e)$_i$

166 (1) 167*(2) 168×(3) 169×(4) 170*(5) 171 (6) 172×(7) 173×(8) 174×(9) 175*(10)
176×(11) 177×(12) 178×(13) 179×(14) 180×(15) 181×(16) 182×(17) 183×(18) 184×(19) 185×(20)
186×(21) 187 (22) 188 (23) 189 (24) 190 (25) 191 (26) 192 (27)

III (a, b)$_i$-(c, d, e)$_j$

193 (1, 1; ab aa / ab aa / ab aa)
194 (1, 1; ab aa / ab bb / ab aa)
195 (1, 1; ab ab / ab ab / ab ab)
196 (1, 1′; ab aa / ab aa / ab aa)
197 (1, 1′; ab ab / ab ab / ab ab)
198 (1, 2; ab aa / ab aa / ab aa)
199 (1, 2; ab ab / ab ab / ab ab)
200 (1, 3; ab aa / ab aa / ab aa)
201 (1, 3; ab ab / ab ab / ab ab)
202 (1, 4; ab aa / ab aa / ab aa)
203 (1, 4; ab ab / ab ab / ab ab)
204 (1, 4; ab ab / ab ba / ab ab)
205 (1, 5; ab aa / ab aa / ab aa)
206 (1, 5; ab ab / ab ab / ab ab)
207 (1, 5; ab ab / ab ba / ab ba)
208 (1, 6; ab aa / ab aa / ab aa)
209 (1, 6; ab ab / ab ab / ab ab)
210 (1, 7; ab aa / ab aa / ab aa)
211 (1, 7; ab aa / ab bb / ab aa)
212 (1, 7; ab ab / ab ab / ab ab)
213 (1, 7′; ab aa / ab aa / ab aa)
214 (1, 7′; ab ab / ab ab / ab ab)
215 (2, 1; aa aa / aa aa / aa aa)
216 (2, 1; ab aa / ab aa / ab aa)
217 (2, 1; ab ab / ab ab / ab ab)
218 (2, 1′; ab aa / ab aa / ab aa)
219× (2, 2; aa aa / aa aa / aa aa)
220 (2, 2; aa ab / aa ab / aa ab)
221× (2, 2; ab ab / ab ab / ab ab)
222× (2, 3; aa aa / aa aa / aa aa)
223 (2, 3; aa ab / aa ab / aa ab)
224× (2, 3; ab ab / ab ab / ab ab)
225× (2, 4; aa aa / aa aa / aa aa)
226 (2, 4; aa ab / aa ab / aa ab)
227× (2, 4; ab ab / ab ab / ab ab)
228× (2, 5; aa aa / aa aa / aa aa)
229 (2, 5; aa ab / aa ab / aa ab)
230× (2, 5; ab ab / ab ab / ab ab)
231× (2, 6; aa aa / aa aa / aa aa)
232 (2, 6; aa ab / aa ab / aa ab)
233× (2, 6; ab ab / ab ab / ab ab)
234 (2, 7; aa aa / aa aa / aa aa)
235 (2, 7; ab aa / ab aa / ab aa)
236 (2, 7; ab ab / ab ab / ab ab)
237 (2, 7′; ab aa / ab aa / ab aa)
238 (3, 1; ab ab / ab ab / ab ab)
239× (3, 2; ab ab / ab ab / ab ab)
240× (3, 3; ab ab / ab ab / ab ab)
241× (3, 4; ab ab / ab ab / ab ab)
242× (3, 4; ab ab / ba ba / ab ab)
243× (3, 5; ab ab / ab ab / ab ab)
244× (3, 5; ab ab / ba ba / ba ba)
245× (3, 6; ab ab / ab ab / ab ab)
246 (3, 7; ab ab / ab ab / ab ab)

IV (a, b, c)$_i$-(d, e)$_j$

247 (1, 1; abc aaa / abc aaa)
248 (1, 1; abc aaa / abc bbb)
249 (1, 1; abc aba / abc aba)
250 (1, 1; abc aba / abc cbc)
251 (1, 1; abc abc / abc abc)
252 (1, 1′; abc aaa / abc aaa)
253 (1, 1′; abc aba / abc aba)
254 (1, 1′; abc aba / abc abb)
255 (1, 1′; abc abc / abc abc)
256 (1, 2; abc aaa / abc aaa)
257 (1, 2; abc aaa / abc aab)
258 (1, 2; abc aba / abc aba)
259 (1, 2; abc abc / abc abc)
260 (1, 3; abc aaa / abc aaa)
261 (1, 3; abc aba / abc aba)
262 (1, 3; abc aba / abc bab)
263 (1, 3; abc abc / abc abc)
264 (1, 3; abc abc / abc acb)
265 (2, 1; aaa aaa / aaa aaa)
266 (2, 1; aaa abb / aaa acc)
267 (2, 1; aba aaa / aba aaa)
268 (2, 1; aba aac / aba aac)
269 (2, 1; aba aba / aba aba)
270 (2, 1; abb aaa / abb aaa)
271 (2, 1; abb abb / abb abb)
272 (2, 1; abc aaa / abc aaa)
273 (2, 1; abc aba / abc aba)
274 (2, 1; abc abb / abc abb)
275 (2, 1; abc abb / abc acc)
276 (2, 1; abc abc / abc abc)
277 (2, 1′; aaa aba / aaa abb)
278 (2, 1′; aba aaa / aba aaa)
279 (2, 1′; abb aaa / abb aaa)
280 (2, 1′; abc aaa / abc aaa)
281 (2, 1′; abc aba / abc aba)
282 (2, 1′; abc aba / abc abb)
283 (2, 1′; abc abb / abc abb)
284× (2, 2; aaa aaa / aaa aaa)
285 (2, 2; aaa aaa / aaa aab)
286 (2, 2; aaa aba / aaa aba)
287 (2, 2; aaa abb / aaa abb)
288 (2, 2; aaa abc / aaa abc)
289× (2, 2; aaa aaa / aab aab)
290 (2, 2; aaa abc / aab abc)
291* (2, 2; aac aba / aac aba)
292× (2, 2; aba aba / aba aba)
293 (2, 2; aba abc / aba abc)
294× (2, 2; abb abb / abb abb)

295$\left(2, 2;\ \begin{matrix}abb & abc\\ abb & abc\end{matrix}\right)$	296$^{\times}\left(2, 2;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	297$^{\times}\left(2, 3;\ \begin{matrix}aaa & aaa\\ aaa & aaa\end{matrix}\right)$	298$\left(2, 3;\ \begin{matrix}aaa & aba\\ aaa & aba\end{matrix}\right)$
299$\left(2, 3;\ \begin{matrix}aaa & abb\\ aaa & abb\end{matrix}\right)$	300$\left(2, 3;\ \begin{matrix}aaa & abc\\ aaa & abc\end{matrix}\right)$	301$\left(2, 3;\ \begin{matrix}aaa & abc\\ aaa & acb\end{matrix}\right)$	302*$\left(2, 3;\ \begin{matrix}aac & aba\\ aac & aba\end{matrix}\right)$
303$^{\times}\left(2, 3;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$	304$\left(2, 3;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$	305$^{\times}\left(2, 3;\ \begin{matrix}abb & abb\\ abb & abb\end{matrix}\right)$	306$\left(2, 3;\ \begin{matrix}abb & abc\\ abb & abc\end{matrix}\right)$
307$^{\times}\left(2, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	308$\left(2, 3;\ \begin{matrix}abc & abc\\ abc & acb\end{matrix}\right)$	309$^{\times}\left(2, 3;\ \begin{matrix}abc & abc\\ acb & acb\end{matrix}\right)$	310$\left(3, 1;\ \begin{matrix}aaa & aaa\\ aaa & aaa\end{matrix}\right)$
311$\left(3, 1;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	312$^{\times}\left(3, 2;\ \begin{matrix}aaa & aaa\\ aaa & aaa\end{matrix}\right)$	313$\left(3, 2;\ \begin{matrix}aaa & aaa\\ aaa & aab\end{matrix}\right)$	314$^{\times}\left(3, 2;\ \begin{matrix}aaa & aaa\\ aab & aab\end{matrix}\right)$
315$^{\times}\left(3, 2;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	316$^{\times}\left(3, 3;\ \begin{matrix}aaa & aaa\\ aaa & aaa\end{matrix}\right)$	317$^{\times}\left(3, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	318$\left(4, 1;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$
319$\left(4, 1;\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$	320$\left(4, 1;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	321$\left(4, 1';\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$	322$^{\times}\left(4, 2;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$
323$\left(4, 2;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$	324$^{\times}\left(4, 2;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	325$^{\times}\left(4, 3;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$	326$\left(4, 3;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$
327$^{\times}\left(4, 3;\ \begin{matrix}aba & aba\\ bab & bab\end{matrix}\right)$	328$^{\times}\left(4, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	329$\left(5, 1;\ \begin{matrix}abb & abb\\ abb & abb\end{matrix}\right)$	330$\left(5, 1;\ \begin{matrix}abc & abb\\ abc & abb\end{matrix}\right)$
331$\left(5, 1;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	332$\left(5, 1';\ \begin{matrix}abc & abb\\ abc & abb\end{matrix}\right)$	333$^{\times}\left(5, 2\cdot\ \begin{matrix}abb & abb\\ abb & abb\end{matrix}\right)$	334$\left(5, 2;\ \begin{matrix}abb & abc\\ abb & abc\end{matrix}\right)$
335$^{\times}\left(5, 2\cdot\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	336$^{\times}\left(5, 3;\ \begin{matrix}abb & abb\\ abb & abb\end{matrix}\right)$	337$\left(5, 3;\ \begin{matrix}abb & abc\\ abb & abc\end{matrix}\right)$	338$^{\times}\left(5, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$
339$^{\times}\left(5, 3;\ \begin{matrix}abb & abb\\ baa & baa\end{matrix}\right)$	340$\left(6, 1;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	341$^{\times}\left(6, 2;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	342$^{\times}\left(6, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$
343$\left(7, 1;\ \begin{matrix}aba & aaa\\ aba & aaa\end{matrix}\right)$	344$\left(7, 1;\ \begin{matrix}aba & aaa\\ aba & bbb\end{matrix}\right)$	345$\left(7, 1;\ \begin{matrix}aba & aac\\ aba & aac\end{matrix}\right)$	346$\left(7, 1;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$
347$\left(7, 1;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$	348$\left(7, 1;\ \begin{matrix}aba & bbb\\ aba & bbb\end{matrix}\right)$	349$\left(7, 1;\ \begin{matrix}abc & aaa\\ abc & aaa\end{matrix}\right)$	350$\left(7, 1;\ \begin{matrix}abc & aaa\\ abc & bbb\end{matrix}\right)$
351$\left(7, 1;\ \begin{matrix}abc & aac\\ abc & aac\end{matrix}\right)$	352$\left(7, 1;\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$	353$\left(7, 1;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	354$\left(7, 1;\ \begin{matrix}abc & bbb\\ abc & bbb\end{matrix}\right)$
355$\left(7, 1';\ \begin{matrix}aba & aaa\\ aba & aaa\end{matrix}\right)$	356$\left(7, 1';\ \begin{matrix}aba & aac\\ aba & aac\end{matrix}\right)$	357$\left(7, 1';\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$	358$\left(7, 1';\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$
359$\left(7, 1';\ \begin{matrix}aba & bbb\\ aba & bbb\end{matrix}\right)$	360$\left(7, 1';\ \begin{matrix}abc & aaa\\ abc & aaa\end{matrix}\right)$	361$\left(7, 1';\ \begin{matrix}abc & aac\\ abc & aac\end{matrix}\right)$	362$\left(7, 1';\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$
363$\left(7, 1';\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	364$\left(7, 1';\ \begin{matrix}abc & bbb\\ abc & bbb\end{matrix}\right)$	365$\left(7, 2;\ \begin{matrix}aba & aaa\\ aba & aaa\end{matrix}\right)$	366$\left(7, 2;\ \begin{matrix}aba & aac\\ aba & aac\end{matrix}\right)$
367$\left(7, 2;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$	368$\left(7, 2;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$	369$\left(7, 2;\ \begin{matrix}aba & bbb\\ aba & bbb\end{matrix}\right)$	370$\left(7, 2;\ \begin{matrix}abc & aaa\\ abc & aaa\end{matrix}\right)$
371$\left(7, 2;\ \begin{matrix}abc & aac\\ abc & aac\end{matrix}\right)$	372$\left(7, 2;\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$	373$\left(7, 2;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	374$\left(7, 2;\ \begin{matrix}abc & bbb\\ abc & bbb\end{matrix}\right)$
375$\left(7, 3;\ \begin{matrix}aba & aaa\\ aba & aaa\end{matrix}\right)$	376$\left(7, 3;\ \begin{matrix}aba & aac\\ aba & aac\end{matrix}\right)$	377$\left(7, 3;\ \begin{matrix}aba & aba\\ aba & aba\end{matrix}\right)$	378$\left(7, 3;\ \begin{matrix}aba & aba\\ aba & bab\end{matrix}\right)$
379$\left(7, 3;\ \begin{matrix}aba & abc\\ aba & abc\end{matrix}\right)$	380$\left(7, 3;\ \begin{matrix}aba & bbb\\ aba & bbb\end{matrix}\right)$	381$\left(7, 3;\ \begin{matrix}abc & aaa\\ abc & aaa\end{matrix}\right)$	382$\left(7, 3;\ \begin{matrix}abc & aac\\ abc & aac\end{matrix}\right)$
383$\left(7, 3;\ \begin{matrix}abc & aba\\ abc & aba\end{matrix}\right)$	384$\left(7, 3;\ \begin{matrix}abc & aba\\ abc & bab\end{matrix}\right)$	385$\left(7, 3;\ \begin{matrix}abc & abc\\ abc & abc\end{matrix}\right)$	386$\left(7, 3;\ \begin{matrix}abc & bbb\\ abc & bbb\end{matrix}\right)$

V (a, b, c, d)$_i$–e

387(1; abcd aaaa)	388(1; abcd abaa)	389(1; abcd abab)	390(1; abcd abca)
391(1; abcd abcd)	392*(2; abab aacc)	393(2; abcd aacc)	394*(2; abcd abcd)
395$^{\times}$(3; aaaa aaaa)	396(3; aaaa abaa)	397(3; aaaa abab)	398(3; aaaa abbb)
399(3; aaaa abca)	400(3; aaaa abcb)	401(3; aaaa abcd)	402*(3; aaca abaa)
403(3; aaca abab)	404*(3; aacc abab)	405(3; aacd abaa)	406*(3; aacd abca)
407$^{\times}$(3; abaa abaa)	408(3; abaa abca)	409(3; abaa abcd)	410$^{\times}$(3; abab abab)
411(3; abab abcb)	412(3; abab abcd)	413(3; abad abab)	414$^{\times}$(3; abbb abbb)
415(3; abbb abcb)	416(3; abbb abcd)	417(3; abbd abca)	418*(3; abbd abcb)
419$^{\times}$(3; abca abca)	420(3; abca abcd)	421$^{\times}$(3; abcb abcb)	422(3; abcb abcd)
423(3; abcc abbb)	424$^{\times}$(3; abcd abcd)	425(3; abdd abaa)	426$^{\times}$(4; aaaa aaaa)
427(4; aaaa aaca)	428(4; aaaa aacc)	429$^{\times}$(4; aaca aaca)	430$^{\times}$(4; aacc aacc)
431$^{\times}$(4; abad abad)	432(4; abad abcd)	433$^{\times}$(4; abbd abbd)	434(4; abbd abcd)
435$^{\times}$(4; abcd abcd)	436*(5; aaaa aaaa)	437(5; aaaa abca)	438(5; aaaa abcb)
439*(5; aaca aaad)	440(5; aaca abcd)	441*(5; abad abca)	442(5; abad abcb)

443*(5; abbd abcb)	444*(5; abcd abcd)	445(6; aaaa aaaa)	446(6; aaaa abcc)
447(6; aacc abcd)	448(6; abcd abcd)	449×(7; aaaa aaaa)	450×(7; abcd abcd)
451×(8; aaaa aaaa)	452×(8; abcd abcd)	453×(9; aaaa aaaa)	454×(9; abcd abcd)
455*(10; aaaa aaaa)	456*(10; abcd abcd)	457×(11; aaaa aaaa)	458×(11; abcc abcc)
459(11; abcc abcd)	460×(11; abcd abcd)	461×(12; aaaa aaaa)	462×(12; abcd abcd)
463×(13; abaa abaa)	464(13; abaa abca)	465(13; abaa abcc)	466(13; abaa abcd)
467×(13; abca abca)	468*(13; abca abad)	469(13; abca abcd)	470×(13; abcc abcc)
471(13; abcc abcd)	472×(13; abcd abcd)	473×(14; abab abab)	474(14; abab abad)
475(14; abab abcb)	476(14; abab abcd)	477×(14; abad abad)	478(14; abad abcb)
479(14; abad abcd)	480×(14; abcb abcb)	481(14; abcb abcd)	482×(14; abcd abcd)
483×(15; abbb abbb)	484(15; abbb abcb)	485(15; abbb abcc)	486(15; abbb abcd)
487*(15; abbd abcb)	488×(15; abcb abcb)	489(15; abcb abcd)	490×(15; abcc abcc)
491(15; abcc abcd)	492×(15; abcd abcd)	493×(16; abaa abaa)	494×(16; abcd abcd)
495×(17; abab abab)	496×(17; abcd abcd)	497×(18; abca abca)	498(18; abca abcd)
499×(18; abcd abcd)	500×(19; abcb abcb)	501(19; abcb abcd)	502×(19; abcd abcd)
503×(20; abcd abcd)	504×(21; abcd abcd)	505(22; abca aaaa)	506(22; abca aaad)
507(22; abca abaa)	508(22; abca abad)	509(22; abca abba)	510(22; abca abbd)
511(22; abca abca)	512(22; abca abcd)	513(22; abca bbbb)	514(22; abca bbcb)
515(22; abcd aaaa)	516(22; abcd aaad)	517(22; abcd abaa)	518(22; abcd abad)
519(22; abcd abba)	520(22; abcd abbd)	521(22; abcd abca)	522(22; abcd abcd)
523(22; abcd bbbb)	524(22; abcd bbcb)	525(23; abca aaaa)	526(23; abca abad)
527(23; abca abbd)	528(23; abca bbbb)	529(23; abcb cccc)	530(23; abcd aaca)
531(23; abcd abcd)	532(23; abcd bbcb)	533(24; abcd aacc)	534(24; abcd abcd)
535(25; abaa aaaa)	536(25; abaa aaca)	537(25; abaa aacc)	538(25; abaa aacd)
539(25; abaa abaa)	540(25; abaa abca)	541(25; abaa abcc)	542(25; abaa abcd)
543(25; abaa bbbb)	544(25; abad aaca)	545(25; abad abca)	546(25; abca aaaa)
547(25; abca aaca)	548(25; abca aacd)	549(25; abca abaa)	550(25; abca abca)
551(25; abca abcd)	552(25; abca bbbb)	553(25; abcc aaaa)	554(25; abcc aacc)
555(25; abcc aacd)	556(25; abcc abaa)	557(25; abcc abcc)	558(25; abcc abcd)
559(25; abcc bbbb)	560(25; abcd aaaa)	561(25; abcd aaca)	562(25; abcd aacc)
563(25; abcd aacd)	564(25; abcd abaa)	565(25; abcd abca)	566(25; abcd abcc)
567(25; abcd abcd)	568(25; abcd bbbb)	569(26; abaa aaaa)	570(26; abaa abaa)
571(26; abaa bbbb)	572(26; abcd aacd)	573(26; abcd abcd)	574(27; abab aaaa)
575(27; abab aaca)	576(27; abab aacc)	577(27; abab abab)	578(27; abab abcb)
579(27; abab abcd)	580(27; abad aaaa)	581(27; abad aaca)	582(27; abad abcb)
583(27; abcb aaaa)	584(27; abcb aaca)	585(27; abcb abab)	586(27; abcb abcb)
587(27; abcb abcd)	588(27; abcd aaaa)	589(27; abcd aaca)	590(27; abcd aacc)
591(27; abcd abab)	592(27; abcd abcb)	593(27; abcd abcd)	

VI $\mathbf{a}<^{(\mathbf{b},\,\mathbf{c})_i}_{(\mathbf{d},\,\mathbf{e})_j}$

594(1,1) 595*(1,1′) 596(1,2) 597(1,3) 598×(2,2) 599×(2,3) 600×(3,3)

VII $a<^{(\mathbf{b},\,\mathbf{c},\,\mathbf{d})_i}_{\mathbf{e}}$

601(1) 602×(2) 603×(3) 604×(4) 605×(5) 606×(6) 607(7)

VIII $\overbrace{(\mathbf{a},\,\mathbf{b})<^{(\mathbf{c},\,\mathbf{d})}_{\mathbf{e}}}^{i}$

608(35; abaa aaaa)	609(35; abaa abaa)	610(35; abaa bbbb)	611(36; abab aaaa)
612(36; abab abab)	613(37; abaa aaaa)	614(38; abaa aaaa)	615(38; abaa abaa)
616(38; abaa bbbb)	617(39; abaa aaaa)	618(40; abaa aaaa)	619(41; abbb aaaa)
620(41; abbb abbb)	621(41; abbb bbbb)	622(42; abaa aaaa)	623(43; abab aaaa)
624(44; abbb aaaa)	625(44; abbb abbb)	626(44; abbb bbbb)	627(45; aaaa aaaa)
628(45; aaaa abaa)	629(45; abaa aaaa)	630(45; abaa abaa)	631(46; aaaa aaaa)
632(46; aaaa abaa)	633(46; aaaa abab)	634(46; aaaa abbb)	635(47; aaaa aaaa)
636(47; abaa aaaa)	637(47; abbb aaaa)	638(48; aaaa aaaa)	639×(49; aaaa aaaa)

640 (49 ; aaaa abaa) 641 (49 ; aaaa abab) 642$^{\times}$ (49 ; abaa abaa) 643$^{\times}$ (49 ; abab abab)
644 (50 ; aaaa aaaa) 645 (50 ; abaa aaaa) 646 (50 ; abbb aaaa) 647$^{\times}$ (51 ; aaaa aaaa)
648$^{\times}$ (52 ; aaaa aaaa) 649 (52 ; aaaa abaa) 650$^{\times}$ (52 ; abaa abaa) 651 (53 ; aaaa aaaa)
652 (53 ; aaaa abaa) 653 (53 ; aaaa abbb) 654$^{\times}$ (54 ; aaaa aaaa) 655 (55 ; abaa abaa)
656$^{\times}$ (56 ; abaa abaa) 657$^{\times}$ (57 ; abaa abaa) 658$^{\times}$ (58 ; abab abab)

IX $\overbrace{(a, b, c)<^{(d)}}^{i}_{e}$

659 (59 ; abca aaaa) 660 (59 ; abca abaa) 661 (59 ; abca abba) 662 (59 ; abca abca)
663 (59 ; abca bbbb) 664 (59 ; abca bbcb) 665 (60 ; abca aaca) 666$^{\times}$ (62 ; aaaa aaaa)
667 (62 ; aaaa abaa) 668 (62 ; aaaa abba) 669 (62 ; aaaa abca) 670* (62 ; aaca abaa)
671$^{\times}$ (62 ; abaa abaa) 672 (62 ; abaa abca) 673$^{\times}$ (62 ; abba abba) 674 (62 ; abba abca)
675$^{\times}$ (62 ; abca abca) 676 (63 ; aaaa aaca) 677* (63 ; aaca aaaa) 678 (63 ; aaca aaca)
679* (63 ; abaa aaaa) 680 (63 ; abaa aaca) 681* (63 ; abab aaaa) 682 (63 ; abab aaca)
683 (63 ; abca aaaa) 684 (63 ; abca aaca) 685 (63 ; abcb aaaa) 686 (63 ; abcb aaca)
687* (64 ; abba aaaa) 688* (64 ; abbb aaaa) 689 (64 ; abca aaaa) 690 (65 ; abbb aaaa)
691* (65 ; abca aaaa) 692* (65 ; abcb aaaa) 693$^{\times}$ (66 ; abaa abaa) 694* (67 ; abaa aaca)
695* (67 ; abab aaca) 696* (67 ; abab aacc) 697$^{\times}$ (72 ; aaaa aaaa) 698$^{\times}$ (72 ; abca abca)
699$^{\times}$ (74 ; abaa abaa) 700 (74 ; abaa abca) 701$^{\times}$ (74 ; abca abca) 702* (75 ; abca abaa)
703* (75 ; abcc abaa) 704$^{\times}$ (77 ; abba abba) 705 (77 ; abca abba) 706$^{\times}$ (77 ; abca abca)
707* (78 ; abca abba) 708$^{\times}$ (80 ; abca abca) 709 (81 ; abaa aaaa) 710 (81 ; abca aaaa)
711 (81 ; abca aaca) 712 (81 ; abca abaa) 713 (81 ; abca abca) 714 (81 ; abca bbbb)
715 (82 ; abaa aaaa) 716 (82 ; abaa abaa) 717 (82 ; abaa bbbb) 718 (82 ; abca aaaa)
719 (82 ; abca abaa) 720 (82 ; abca bbbb) 721 (82 ; abcc aaaa) 722 (82 ; abcc abaa)
723 (82 ; abcc bbbb) 724 (83 ; abaa aaaa) 725 (83 ; abab bbbb) 726 (83 ; abca aaaa)
727 (83 ; abca aaca) 728 (83 ; abcb bbbb) 729 (84 ; abaa aaaa) 730 (84 ; abab bbbb)
731 (84 ; abca aaaa) 732 (84 ; abcb bbbb) 733 (84 ; abcc aaaa) 734 (85 ; abab aaaa)
735 (85 ; abab bbbb) 736 (85 ; abcb aaaa) 737 (85 ; abcb aaca) 738 (85 ; abcb abab)
739 (85 ; abcb abcb) 740 (85 ; abcb bbbb)

X a–b–(c, d, e)$_i$

741 (1) 742$^{\times}$ (2) 743$^{\times}$ (3) 744$^{\times}$ (4) 745$^{\times}$ (5) 746$^{\times}$ (6) 747 (7)

XI a–$\overbrace{(b, c)}^{i}$–(d, e)

748 (35) 749 (36) 750 (37) 751 (38) 752 (39) 753 (40) 754 (41) 755 (42) 756 (43) 757 (44)
758 (45) 759 (46) 760 (47) 761 (48) 762$^{\times}$ (49) 763 (50) 764$^{\times}$ (51) 765$^{\times}$ (52) 766 (53) 767$^{\times}$ (54)
768 (55) 769$^{\times}$ (56) 770$^{\times}$ (57) 771$^{\times}$ (58)

XII a–$\overbrace{(b, c, d)}^{i}$–(e)

772 (59) 773 (60) 774 (61) 775$^{\times}$ (62) 776 (63) 777 (64) 778 (65) 779$^{\times}$ (66) 780* (67) 781 (68)
782$^{\times}$ (69) 783 (70) 784$^{\times}$ (71) 785$^{\times}$ (72) 786$^{\times}$ (73) 787$^{\times}$ (74) 788 (75) 789$^{\times}$ (76) 790$^{\times}$ (77) 791 (78)
792$^{\times}$ (79) 793$^{\times}$ (80) 794 (81) 795 (82) 796 (83) 797 (84) 798 (85) 799 (86) 800 (87) 801 (88)
802 (89) 803 (90)

XIII (a, b)$_i$–c–(d, e)$_j$

804 $\left(1, 1;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ 805 $\left(1, 1;\ \begin{matrix} ab & bb \\ ab & ab \\ ab & ab \end{matrix}\right)$ 806 $\left(1, 1;\ \begin{matrix} ab & bb \\ ab & bb \\ ab & bb \end{matrix}\right)$ 807 $\left(1, 1';\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ 808 $\left(1, 1';\ \begin{matrix} ab & bb \\ ab & ab \\ ab & ab \end{matrix}\right)$

809 $\left(1, 1';\ \begin{matrix} ab & bb \\ ab & bb \\ ab & bb \end{matrix}\right)$ 810 $\left(1, 2;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ 811 $\left(1, 2;\ \begin{matrix} ab & bb \\ ab & ab \\ ab & ab \end{matrix}\right)$ 812 $\left(1, 2;\ \begin{matrix} ab & bb \\ ab & bb \\ ab & bb \end{matrix}\right)$ 813 $\left(1, 3;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$

814 $\left(1, 3;\ \begin{matrix} ab & bb \\ ab & ab \\ ab & ab \end{matrix}\right)$ 815 $\left(1, 3;\ \begin{matrix} ab & bb \\ ab & bb \\ ab & bb \end{matrix}\right)$ 816 $\left(2, 1;\ \begin{matrix} aa & aa \\ aa & aa \\ aa & aa \end{matrix}\right)$ 817 $\left(2, 1;\ \begin{matrix} aa & aa \\ aa & ab \\ aa & ab \end{matrix}\right)$ 818 $\left(2, 1;\ \begin{matrix} aa & ab \\ aa & ab \\ aa & ab \end{matrix}\right)$

819 $\left(2, 1;\ \begin{matrix} aa & aa \\ ab & aa \\ ab & aa \end{matrix}\right)$ 820 $\left(2, 1;\ \begin{matrix} aa & aa \\ ab & ab \\ ab & ab \end{matrix}\right)$ 821 $\left(2, 1;\ \begin{matrix} aa & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ 822 $\left(2, 1;\ \begin{matrix} ab & aa \\ ab & aa \\ ab & aa \end{matrix}\right)$ 823 $\left(2, 1;\ \begin{matrix} ab & aa \\ ab & ab \\ ab & ab \end{matrix}\right)$

$824\left(2, 1;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $825^{\times}\left(2, 2;\ \begin{matrix} aa & aa \\ aa & aa \\ aa & aa \end{matrix}\right)$ $826\left(2, 2;\ \begin{matrix} aa & aa \\ aa & ab \\ aa & ab \end{matrix}\right)$ $827\left(2, 2;\ \begin{matrix} aa & ab \\ aa & ab \\ aa & ab \end{matrix}\right)$ $828^{\times}\left(2, 2;\ \begin{matrix} aa & aa \\ ab & ab \\ ab & ab \end{matrix}\right)$

$829\left(2, 2;\ \begin{matrix} aa & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $830^{\times}\left(2, 2;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $831^{\times}\left(2, 3;\ \begin{matrix} aa & aa \\ aa & aa \\ aa & aa \end{matrix}\right)$ $832\left(2, 3;\ \begin{matrix} aa & aa \\ aa & ab \\ aa & ab \end{matrix}\right)$ $833\left(2, 3;\ \begin{matrix} aa & ab \\ aa & ab \\ aa & ab \end{matrix}\right)$

$834^{\times}\left(2, 3;\ \begin{matrix} aa & aa \\ ab & ab \\ ab & ab \end{matrix}\right)$ $835\left(2, 3;\ \begin{matrix} aa & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $836^{\times}\left(2, 3;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $837\left(3, 1;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$ $838^{\times}\left(3, 2;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$

$839^{\times}\left(3, 3;\ \begin{matrix} ab & ab \\ ab & ab \\ ab & ab \end{matrix}\right)$

XIV $\overbrace{(\mathbf{a}, \mathbf{b})\text{-}(\mathbf{c}, \mathbf{d})}^{i}\text{-}\mathbf{e}$

840 (35 ; abcd aacc)	841 (35 ; abcd aacd)	842 (35 ; abcd abcc)	843 (35 ; abcd abcd)
844 (36 ; abcd aacc)	845 (36 ; abcd abcd)	846 (37 ; abcd abcc)	847 (37 ; abcd abcd)
848 (38 ; abcc aacd)	849 (38 ; abcc abcd)	850 (38 ; abcd aacd)	851 (38 ; abcd abcd)
852 (39 ; abcc abcd)	853 (39 ; abcd abcd)	854 (40 ; abcc abcc)	855 (40 ; abcc abcd)
856 (40 ; abcd abcc)	857 (40 ; abcd abcd)	858 (41 ; abcc abcc)	859 (41 ; abcc abcd)
860 (41 ; abcc bbcc)	861 (41 ; abcc bbcd)	862 (41 ; abcd abcc)	863 (41 ; abcd abcd)
864 (41 ; abcd bbcc)	865 (41 ; abcd bbcd)	866 (42 ; abcd abcd)	867 (43 ; abcd abcd)
868 (44 ; abcd abcd)	869 (44 ; abcd bbcd)	870 (45 ; aacd aacc)	871 (45 ; aacd aacd)
872 (45 ; aacd abcc)	873 (45 ; aacd abcd)	874 (45 ; abcd aacc)	875 (45 ; abcd aacd)
876 (45 ; abcd abcc)	877 (45 ; abcd abcd)	878 (46 ; aacd aacc)	879 (46 ; abcd aacd)
880 (46 ; abcd abcc)	881 (46 ; abcd abcd)	882 (47 ; aacd abcc)	883 (47 ; aacd abcd)
884 (47 ; abcd abcc)	885 (47 ; abcd abcd)	886 (48 ; abcd abcc)	887 (48 ; abcd abcd)
888$^{\times}$(49 ; aacc aacc)	889 (49 ; aacc aacd)	890 (49 ; aacc abcc)	891 (49 ; aacc abcd)
892$^{\times}$(49 ; aacd aacd)	893 (49 ; aacd abcc)	894 (49 ; aacd abcd)	895$^{\times}$(49 ; abcc abcc)
896 (49 ; abcc abcd)	897$^{\times}$(49 ; abcd abcd)	898 (50 ; aacc abcc)	899 (50 ; aacc abcd)
900 (50 ; aacd abcc)	901 (50 ; aacd abcd)	902 (50 ; abcc abcc)	903 (50 ; abcc abcd)
904 (50 ; abcd abcc)	905 (50 ; abcd abcd)	906$^{\times}$(51 ; abcc abcc)	907 (51 ; abcc abcd)
908$^{\times}$(51 ; abcd abcd)	909$^{\times}$(52 ; aacd aacd)	910 (52 ; aacd abcd)	911$^{\times}$(52 ; abcd abcd)
912 (53 ; abcd aacd)	913 (53 ; abcd abcd)	914$^{\times}$(54 ; abcd abcd)	915 (55 ; abcd abcc)
916 (55 ; abcd abcd)	917$^{\times}$(56 ; abcc abcc)	918 (56 ; abcc abcd)	919$^{\times}$(56 ; abcd abcd)
920$^{\times}$(57 ; abcd abcd)	921$^{\times}$(58 ; abcd abcd)		

XV $\overbrace{(\mathbf{a}, \mathbf{b}, \mathbf{c})\text{-}(\mathbf{d})}^{i}\text{-}\mathbf{e}$

922 (59 ; abcd aaad)	923 (59 ; abcd abad)	924 (59 ; abcd abbd)	925 (59 ; abcd abcd)
926 (60 ; abcd abad)	927 (60 ; abcd abcd)	928 (61 ; abcd abcd)	929$^{\times}$(62 ; aaad aaad)
930 (62 ; aaad abad)	931 (62 ; aaad abbd)	932 (62 ; aaad abcd)	933* (62 ; aacd abad)
934$^{\times}$(62 ; abad abad)	935 (62 ; abad abcd)	936$^{\times}$(62 ; abbd abbd)	937 (62 ; abbd abcd)
938$^{\times}$(62 ; abcd abcd)	939 (63 ; aaad abad)	940 (63 ; aaad abcd)	941 (63 ; aacd abad)
942 (63 ; aacd abcd)	943 (63 ; abad abad)	944 (63 ; abad abcd)	945 (63 ; abcd abad)
946 (63 ; abcd abcd)	947 (64 ; aaad abbd)	948 (64 ; aaad abcd)	949 (64 ; abbd abbd)
950 (64 ; abbd abcd)	951 (64 ; abcd abbd)	952 (64 ; abcd abcd)	953 (65 ; aaad abcd)
954 (65 ; abad abcd)	955 (65 ; abbd abcd)	956 (65 ; abcd abcd)	957$^{\times}$(66 ; aacd aacd)
958 (66 ; aacd abcd)	959$^{\times}$(66 ; abcd abcd)	960* (67 ; aacd abad)	961 (67 ; aacd abcd)
962* (67 ; abcd abcd)	963 (68 ; aacd abcd)	964 (68 ; abcd abcd)	965$^{\times}$(69 ; abbd abbd)
966 (69 ; abbd abcd)	967$^{\times}$(69 ; abcd abcd)	968 (70 ; abcd abcd)	969 (70 ; abcd accd)
970$^{\times}$(71 ; abcd abcd)	971$^{\times}$(72 ; aaad aaad)	972$^{\times}$(72 ; abcd abcd)	973$^{\times}$(73 ; abcd abcd)
974$^{\times}$(74 ; abad abad)	975 (74 ; abad abcd)	976$^{\times}$(74 ; abcd abcd)	977 (75 ; abad abcd)
978 (75 ; abcd abcd)	979$^{\times}$(76 ; abcd abcd)	980$^{\times}$(77 ; abbd abbd)	981 (77 ; abbd abcd)
982$^{\times}$(77 ; abcd abcd)	983 (78 ; abbd abcd)	984 (78 ; abcd abcd)	985$^{\times}$(79 ; abcd abcd)
986$^{\times}$(80 ; abcd abcd)	987 (81 ; abad aaad)	988 (81 ; abad aacd)	989 (81 ; abad abad)
990 (81 ; abad abcd)	991 (81 ; abcd aaad)	992 (81 ; abcd aacd)	993 (81 ; abcd abad)
994 (81 ; abcd abcd)	995 (82 ; abad aacd)	996 (82 ; abad abcd)	997 (82 ; abcd aacd)

998(82; abcd abcd) 999(83; abad abad) 1000(83; abad abcd) 1001(83; abcd abad)
1002(83; abcd abcd) 1003(84; abad abcd) 1004(84; abcd abcd) 1005(85; abad abad)
1006(85; abad abcd) 1007(85; abad bbbd) 1008(85; abcd abad) 1009(85; abcd abcd)
1010(85; abcd bbbd) 1011(86; abcd aaad) 1012(86; abcd aacd) 1013(86; abcd abad)
1014(86; abcd abcd) 1015(87; abcd aacd) 1016(87; abcd abcd) 1017(88; abcd abad)
1018(88; abcd abcd) 1019(89; abcd abcd) 1020(90; abcd abad) 1021(90; abcd abcd)
1022(90; abcd bbbd)

XVI a< (b, c)$_i$, d, e

1023(1) 1024×(2) 1025×(3)

XVII $\overbrace{\text{(a, b)<(c), (d)}}^{i}$, e

1026(94; abaa aaaa) 1027(94; abba bbbb) 1028(95; abbb aaaa) 1029(95; abbb bbbb)
1030×(97; aaaa aaaa) 1031(97; aaaa abaa) 1032*(98; abaa aaaa) 1033*(98; abba aaaa)
1034×(101; aaaa aaaa) 1035×(102; abaa abaa)

XVIII a< b-(c, d)$_i$, e

1036(1) 1037×(2) 1038×(3)

XIX a< (b, c)$_i$, d-e

1039(1) 1040×(2) 1041×(3)

XX a< $\overbrace{\text{(b, c)-(d)}}^{i}$, e

1042(11) 1043(12) 1044×(13) 1045(14) 1046×(15) 1047×(16)

XXI $\overbrace{\text{(a, b)<(c)-(d)}}^{i}$, e

1048(112; abaa aaaa) 1049(113; abbb aaaa) 1050(113; abbb abbb) 1051(113; abbb bbbb)
1052(114; abba aaaa) 1053(114; abbb bbbb) 1054×(115; aaaa aaaa) 1055(115; aaaa abaa)
1056×(115; abaa abaa) 1057(116; aaaa aaaa) 1058(116; abaa aaaa) 1059(116; abab aaaa)
1060×(117; aaaa aaaa) 1061(118; aaaa aaaa) 1062(118; abaa aaaa) 1063(118; abbb aaaa)
1064(119; aaaa aaaa) 1065×(120; aaaa aaaa) 1066×(121; abaa abaa)

XXII a< b, c >(d, e)$_i$

1067(1) 1068×(2) 1069×(3)

XXIII a< $\overbrace{\text{(b, c)>(d)}}^{i}$, e

1070(11) 1071(12) 1072×(13) 1073(14) 1074×(15) 1075×(16)

XXIV $\overbrace{\text{(a, b)<(c), (d)}}^{i}$ >e

1076(94; abcd abcd) 1077(95; abcd abcd) 1078(95; abcd bbcd) 1079(96; abcd abcd)
1080×(97; aacd aacd) 1081(97; abcd aacd) 1082×(97; abcd abcd) 1083(98; aacd abcd)
1084(98; abcd abcd) 1085*(99; abcd abcd) 1086*(100; abcd abcd) 1087×(101; abcd abcd)
1088×(102; abcd abcd)

XXV a-b< (c,d)$_i$, e

1089(1) 1090×(2) 1091×(3)

XXVI $a\text{-}\overbrace{(b,\ c)<^{(d)}_{(e)}}^{i}$

1092(94) 1093(95) 1094(96) 1095×(97) 1096(98) 1097*(99) 1098*(100) 1099×(101) 1100×(102)

XXVII $\overbrace{(a,\ b)\text{-}(c)<^{(d)}_{e}}^{i}$

1101(112; abcc abcc) 1102(113; abcc abcc) 1103(113; abcc bbcc) 1104×(115; aacc aacc)
1105(115; abcc aacc) 1106×(115; abcc abcc) 1107*(116; abcc aacc) 1108(118; abcc abcc)
1109(118; aacc abcc) 1110×(120; abcc abcc) 1111×(121; abcc abcc)

XXVIII $a\text{-}b\text{-}c\text{-}(d,\ e)_i$

1112(103) 1113×(104) 1114×(105)

XXIX $a\text{-}\overbrace{(b)\text{-}(c,\ d)\text{-}(e)}^{i}$

1115(106) 1116(107) 1117×(108) 1118(109) 1119×(110) 1120×(111)

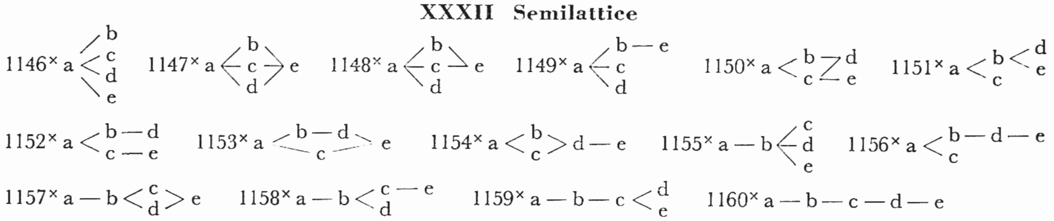
XXX $a\text{-}\overbrace{(b,\ c)\text{-}(d)\text{-}(e)}^{i}$

1121(112) 1122(113) 1123(114) 1124×(115) 1125(116) 1126×(117) 1127(118) 1128(119)
1129×(120) 1130×(121)

XXXI $\overbrace{(a,\ b)\text{-}(c)\text{-}(d)}^{i}\text{-}e$

1131(112; abcd abcd) 1132(113; abcd abcd) 1133(113; abcd bbcd) 1134(114; abcd abcd)
1135×(115; aacd aacd) 1136(115; abcd aacd) 1137×(115; abcd abcd) 1138(116; aacd abcd)
1139(116; abcd abcd) 1140×(117; abcd abcd) 1141(118; aacd abcd) 1142(118; abcd abcd)
1143(119; abcd abcd) 1144×(120; abcd abcd) 1145×(121; abcd abcd)

XXXII Semilattice



List 7 Motzkin's Numbers for Our Numbers

	0	1	2	3	4	5	6	7	8	9
0		0994	0654	0655	0000	0005	0034	0019	0037	0020
10	0049	0047	0017	0048	0038	0054	0018	0001	0003	0004
20	0007	0008	0025	0117	0120	0123	0126	0203	0015	0016
30	0033	0036	0046	0021	0071	0022	0076	0039	0041	0023
40	0077	0052	0024	0078	0050	0053	0072	0040	0081	0051
50	0042	0082	0084	0132	0134	0115	0136	0198	0118	0121
60	0145	0124	0133	0141	0135	0147	0116	0137	0199	0119
70	0138	0140	0122	0143	0146	0200	0139	0142	0201	0125
80	0144	0148	0202	0221	0223	0226	0224	0227	0228	0238
90	0240	0244	0247	0248	0249	0253	0127	0995	1062	1119
100	1152	0996	1064	1063	1120	0998	1121	1123	0999	1122
110	1003	1129	1124	1004	1125	1009	1065	1126	1011	1127
120	1012	1128	1005	1130	1109	1144	1153	1154	1110	1155
130	1149	1157	1158	1148	1156	1159	0404	0580	0660	1075
140	1076	1143	0114	0197	0391	0392	0574	0578	0403	0075
150	0155	0151	0083	0251	0239	0243	0252	0014	0112	0045
160	0241	0152	0153	0225	0250	0154	0935	0968	0834	0835
170	0837	0838	0843	0844	0849	0851	0853	0845	0936	0938
180	0962	0963	0982	0972	0984	0990	0989	0872	0887	0969
190	0842	0852	0870	0810	0811	0961	0827	0986	0779	0937
200	0780	0939	0813	0971	0973	0825	0983	0985	0831	0992

	0	1	2	3	4	5	6	7	8	9
210	0785	0786	0944	0812	0970	0313	0383	0674	0370	0294
220	0355	0661	0295	0356	0662	0321	0372	0675	0330	0379
230	0677	0337	0386	0678	0298	0371	0665	0359	1057	1044
240	1045	1058	1099	1060	1141	1061	1048	0766	0767	0823
250	0824	0981	0772	0828	0829	0987	0760	0762	0814	0974
260	0773	0830	0832	0988	0991	0064	0100	0091	0213	0208
270	0101	0583	0108	0219	0589	0593	0596	0093	0089	0098
280	0107	0218	0573	0588	0062	0065	0087	0096	0105	0131
290	0150	0211	0206	0216	0581	0586	0594	0069	0092	0102
300	0109	0110	0215	0210	0220	0585	0590	0597	0598	0653
310	0234	0601	0232	0235	0246	0599	0236	0602	1017	1023
320	1036	1022	1015	1020	1034	1019	1024	1070	1037	1096
330	1097	1103	1102	1089	1090	1098	1104	1105	1106	1136
340	1117	1115	1118	0284	0287	0351	0325	0378	0399	0621
350	0625	0638	0630	0644	0650	0291	0352	0335	0384	0400
360	0626	0640	0632	0645	0651	0281	0348	0322	0373	0397
370	0619	0628	0636	0642	0648	0292	0353	0336	0339	0385
380	0401	0627	0641	0633	0634	0646	0652	0732	0774	0826
390	0833	0993	1086	1087	1107	0002	0006	0009	0011	0010
400	0012	0013	0027	0029	0158	0031	0162	0026	0028	0032
410	0156	0159	0160	0157	0405	0407	0408	0163	0410	0161
420	0164	0409	0411	0406	0432	0030	0035	0043	0044	0165
430	0166	0167	0168	0412	0413	0433	0056	0073	0074	0079
440	0080	0436	0437	0438	0439	0057	0149	0204	0440	0128
450	0458	0129	0459	0222	0468	0230	0471	0245	0474	0475
460	0476	0130	0460	0997	1000	1001	1002	1006	1007	1008
470	1025	1026	1027	1066	1068	1071	1073	1069	1072	1074
480	1077	1078	1079	1131	1132	1133	1134	1137	1135	1138
490	1139	1140	1142	1010	1028	1067	1080	1111	1112	1113
500	1145	1146	1147	1151	1150	0273	0293	0346	0354	0334
510	0382	0342	0390	0396	0402	0563	0564	0567	0568	0565
520	0569	0566	0570	0571	0572	0575	0576	0577	0579	0659
530	0656	0657	0658	1088	1108	0061	0086	0095	0104	0070
540	0189	0103	0111	0113	0192	0193	0188	0094	0194	0190
550	0191	0455	0196	0425	0427	0429	0426	0428	0430	0431
560	0449	0451	0453	0195	0450	0452	0454	0456	0457	0231
570	0237	0242	0472	0473	0270	0348	0347	0331	0338	0387
580	0272	0345	0381	0393	0394	0333	0340	0388	0549	0552
590	0550	0553	0551	0554	0741	0749	0738	0752	0736	0750
600	0769	0710	0697	0698	0711	0716	0721	0701	0692	0755
610	0728	0693	0758	0715	0694	0768	0729	0719	0712	0726
620	0753	0689	0720	0722	0730	0770	0695	0259	0288	0278
630	0498	0267	0537	0538	0541	0266	0525	0528	0605	0257
640	0276	0279	0495	0496	0264	0523	0526	0603	0261	0289
650	0508	0268	0539	0542	0606	1041	1039	1042	1084	0687
660	0696	0718	0724	0725	0731	0771	0055	0058	0059	0060
670	0170	0169	0171	0414	0415	0434	0085	0181	0182	0175
680	0176	0177	0178	0185	0441	0186	0442	0418	0419	0422
690	0423	0446	0447	0461	0462	0463	0464	0229	0469	1013
700	1014	1029	1031	1032	1081	1082	1083	1085	1114	0256
710	0482	0484	0483	0485	0486	0263	0290	0344	0515	0516
720	0517	0518	0519	0520	0262	0332	0509	0510	0514	0269
730	0380	0540	0548	0543	0271	0395	0555	0557	0556	0558
740	0559	0923	0910	0911	0924	0929	0932	0914	0905	0906
750	0928	0907	0930	0925	0902	0931	0933	0908	0859	0867
760	0866	0890	0857	0864	0888	0861	0868	0891	0955	0951
770	0958	0967	0900	0909	0934	0836	0839	0840	0841	0846
780	0847	0848	0873	0874	0875	0850	0876	0940	0942	0946
790	0964	0965	0966	0978	0856	0863	0862	0869	0871	0882
800	0884	0883	0885	0886	0956	0805	0797	0957	0806	0799
810	0952	0803	0795	0959	0807	0800	0309	0318	0366	0319
820	0610	0616	0367	0617	0671	0307	0316	0364	0608	0614
830	0669	0311	0320	0368	0611	0618	0672	1054	1052	1055
840	0793	0801	0802	0808	0794	0809	0950	0960	0815	0819
850	0816	0820	0975	0976	0941	0945	0943	0947	0783	0784
860	0781	0782	0789	0790	0787	0788	0977	0979	0821	0817
870	0305	0312	0314	0315	0326	0327	0504	0505	0374	0375
880	0544	0545	0363	0369	0535	0536	0668	0673	0296	0297
890	0299	0300	0301	0302	0303	0499	0500	0501	0357	0358
900	0360	0361	0529	0530	0531	0532	0663	0664	0666	0323

	0	1	2	3	4	5	6	7	8	9
910	0328	0511	0376	0546	0676	1051	1056	1046	1047	1049
920	1059	1101	0761	0763	0764	0765	0818	0822	0980	0063
930	0066	0067	0068	0173	0172	0174	0416	0417	0435	0088
940	0090	0183	0184	0179	0180	0205	0443	0097	0099	0420
950	0421	0444	0445	0106	0187	0424	0448	0207	0209	0465
960	0212	0214	0466	0217	0467	0582	0584	0591	0592	0587
970	0595	0233	0470	0600	1016	1018	1030	1021	1033	1035
980	1091	1092	1094	1093	1095	1100	1116	0282	0285	0283
990	0286	0491	0492	0493	0494	0349	0521	0350	0522	0324
1000	0329	0512	0513	0377	0547	0341	0389	0398	0560	0561
1010	0562	0620	0622	0623	0624	0637	0639	0629	0631	0643
1020	0635	0647	0649	0683	0681	0684	0685	0717	0686	0680
1030	0254	0255	0479	0480	0477	1038	0707	0705	0708	0740
1040	0737	0751	0704	0709	0699	0700	0702	0713	0714	0727
1050	0754	0690	0723	0691	0258	0277	0497	0260	0488	0489
1060	0478	0265	0527	0524	0481	0604	1040	0777	0775	0778
1070	0747	0748	0742	0743	0744	0756	0757	0735	0734	0759
1080	0274	0275	0487	0280	0490	0612	0613	0607	1043	0896
1090	0894	0897	0898	0893	0899	0854	0855	0879	0880	0877
1100	0948	0949	0792	0791	0304	0306	0502	0506	0533	0362
1110	0667	1050	0920	0918	0921	0922	0917	0912	0913	0915
1120	0926	0927	0903	0904	0858	0860	0878	0865	0881	0889
1130	0953	0954	0798	0796	0804	0308	0310	0503	0317	0507
1140	0609	0365	0534	0615	0670	1053	0679	0733	0688	0682
1150	0745	0703	0739	0746	0776	0892	0706	0901	0895	0916
1160	0919									

List 8 Structure of u.z. and c.d.u.

u.z.

left ordering	right ordering	No.	left ordering	right ordering	No.
a<(b, c, d)	a<(b, c, d)	4	a<(b, c, d), b, c>e	a<(b—d, c—e)	5
a<(b—d, c—e)	a<(b—d, c—e)	6	a<(b—d, c—e)	a<(b—e, c—d)	7
a<(b—d, c—e)	a<(b, c) Z (d, e)	8	a<(b—d, c—e)	a<(b, c) Z (e, d)	9
a<(b—d, c—e)	a<(d, c) X (d, e)	10	a<(b, c) Z (d, e)	a<(b, c) Z (d, e)	11, 12
a<(b, c) Z (d, e)	a<(b, c) Z (e, d)	13, 14	a<(b, c) X (d, e)	a<(b, c) X (d, e)	15
a<(b—d—e, c)	a<(b—d—e, c)	16	a<(b—e, c, d)	a<(b—e, c, d)	17
a<(b—e, c, d)	a<(b—d, c, e)	18	a<(b—e, c, d)	a<(b<(d, e), c)	19
a<(b—e, c, d)	a<(b<(c, d), e)	20	a<(b—e, c, d)	a—b<(c, d, e)	21
a<(b—c—e, d)	a—b<(c—e, d)	22	a—b<(c—e, d)	a—b<(c—e, d)	23∼26
a—b—c<(d, e)	a—b—c<(d, e)	27	a<(b<(d, e), c)	a<(b<(d, e), c)	28∼32
a<(b<(d, e), c)	a<(b<(c, e), d)	33∼38	a<(b<(d, e), c)	a—b<(c, d, e)	39∼52
a—b<(c, d, e)	a—b<(c, d, e)	53∼95	a—b—c—d—e	a—b—c—d—e	96

c. d. u.

greatest c-decomp.	No.	greatest c-decomp.	No.
2_2	136~139	4_3	140
5_3	141	2_3	142~154
3_3	155, 156	3_4	157, 158
4_4	159, 160	7_4	161
8_4	162	9_4	163
11_4	164	12_4	165