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EVALUATION OF SOME ω_n^2 DISTRIBUTION

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Some numerical computations of ω_n^2-distribution[1] for the cases $n=2$ and 9 were executed. We have selected them, one as extreme and the other as intermediate, in order to grasp the general feature. We had to evaluate

$$\Phi(\omega_n^2)=1-\frac{\sqrt{n}}{\pi}\sum_{\kappa=1}^{p}(-1)^{\kappa-1}\int_{\varsigma_{2\kappa-1}}^{\varsigma_{2\kappa}}\frac{\exp\left\{-\frac{1}{2}n^2\omega_n^2\varsigma\right\}d\varsigma}{\sqrt{(1-)^n P_{n-1}(\varsigma)\,\varsigma}}, \qquad (1)$$

where $p=\frac{n-1}{2}$ or $\frac{n}{2}$ according as $n=$ odd or even, and ς_ν $(\nu=1,2,\cdots)$ are the roots of $P_{n-1}(\varsigma)=\sum_{l=0}^{n-1}(-1)^l\binom{2n-1-l}{l}\varsigma^{n-1-l}=0$, but if $n=\text{even}=2p$ the last ς_{2p} should be reckoned as ∞.

For convenience of calculation, we put $\varsigma=\frac{1}{2}\left[\varsigma_{2\kappa}+\varsigma_{2\kappa-1}+(\varsigma_{2\kappa}-\varsigma_{2\kappa-1})\sin\frac{\pi}{2}t\right]$, so that (1) becomes

$$\Phi(\omega_n^2)=1-\frac{\sqrt{n}}{2}\sum_{\kappa=1}^{p}(-1)^{\kappa-1}\int_{-1}^{1}\frac{\exp\left\{-\frac{1}{2}n^2\omega_n^2\varsigma\right\}dt}{\varsigma\sqrt{\prod_{\nu\neq\kappa}(\varsigma-\varsigma_{2\nu-1})(\varsigma-\varsigma_{2\nu})}}. \qquad (2)$$

and that will do when $n=2p+1$. However if $n=2p$, the last factor of $\prod_{\nu\neq\kappa}$ for every summand in (2) shoul be single $\varsigma_{2p-1}-\varsigma$, and besides in the last summand of (1), we must set $\varsigma=\varsigma_{2p-1}\sec^2\frac{\pi}{2}t$, so that the corresponding summand in (2) becomes

$$(-1)^{p-1}\frac{\sqrt{n}}{2}\int_{-1}^{1}\frac{\exp\{-2p^2\omega_n^2\}\,dt}{\sqrt{\varsigma_{2p-1}}\sqrt{\prod_{\nu=1}^{2p-2}(\varsigma-\varsigma_\nu)}}.$$

Now that every summand takes form $\frac{1}{2}\int_{-1}^{1}f_\kappa(t)\,dt$, after Gauss, we may equalize it to $\sum R_\nu y_\nu$ with $y_\nu=f(t_\nu,\omega_n^2,\kappa)$ and thus we can compute

$$\Phi(\omega_n^2)=\int_0^{\omega_n^2}\varphi(\omega_n^2)\,d\omega_n^2=1-\sqrt{n}\sum_{\kappa=1}^{p}(-1)^{\kappa-1}\sum_{\nu=1}^{5}R_\nu y_\nu(t_\nu,\omega_n^2,\kappa).$$

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[1] Cf. Y. Watanabe, On the ω^2 Distribution, this Journal, Vol. 2, pp. 21-30.

Owing to troublesomeness, much with even n, only the cases $n = 2$ and $n = 9$ were treated, the results of which are given in the following Tables.[2)]

Table of $\Phi(\omega_2^2) = \int_0^{\omega_2^2} \varphi(\omega_2^2)\, d\omega_2^2$

ω_2^2	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	$\Phi=0$	.2528	.3329	.3829	.4254	.4640	.4996	.5084	.5630	.5912
0.1	.6117	.6364	.6594	.6806	.7004	.7188	.7358	.7417	.7473	.7801
0.2	.7950	.8069	.8179	.8281	.8377	.8466	.8560	.8579	.8627	.8769
0.3	.8834	.8896	.8952	.9005	.9049	.9103	.9148	.9174	.9199	.9267
0.4	.9302	.9335	.9367	.9396	.9424	.9451	.9477	.9494	.9511	.9545
0.5	.9542	.9561	.9579	.9597	.9613	.9629	.9644	.9658	.9671	.9686
0.6	.9722	.9734	.9745	.9755	.9766	.9776	.9785	.9793	.9801	.9811
0.7	.9819	.9826	.9834	.9840	.9847	.9853	.9859	.9865	.9870	.9876
0.8	.9881	.9886	.9890	.9895	.9899	.9903	.9907	.9911	.9914	.9918
0.9	.9921	.9924	.9927	.9930	.9933	.9936	.9938	.9941	.9943	.9945
ω_2^2	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	∞
Φ	.9947	.9965	.9977	.9984	.9990	.9992	.9995	9997	.9998	1

Table of $\Phi(\omega_9^2) = \int_0^{\omega_9^2} \varphi(\omega_9^2)\, d\omega_9^2$

ω_9^2	00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	$\Phi=0$	.0060	.0114	.0504	.1025	.1562	.2113	.2723	.3226	.3813
0.1	4267	.4713	.5122	.5495	.5835	.6176	.6428	.6691	.6923	.7145
0.2	.7339	.7522	.7690	.7845	.7987	.8120	.8242	.8355	.8459	.8558
0.3	.8608	8727	.8774	.8849	.8919	.8987	.9045	.9102	.9156	.9206
0.4	.9274	.9316	.9356	.9393	.9429	.9462	.9492	.9522	.9549	.9574
0.5	.9599	.9622	.9643	.9663	.9680	.9700	.9715	.9732	.9747	.9761
0.6	.9775	.9787	.9799	.9810	.9820	.9830	.9839	.9848	.9856	.9864
0.7	.9871	.9878	.9885	.9891	.9897	.9902	.9908	.9912	.9917	.9922
0.8	.9926	.9930	.9933	.9937	.9940	.9944	.9947	.9949	.9952	.9955
0.9	.9957	.9959	.9961	.9963	9965	.9967	.9969	.9970	.9973	.9973
ω_9^2	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	∞
Φ	.9975	.9985	.9991	.9995	.9997	.9998	.9999	.9999	1.0000	1

On comparing the present Tables with that for the case $n = \infty$,[3)] we see that their difference is not so remarkable, except when the argument ω^2 is rather small. In fact, the values of $\Phi(\omega_2^2)$ are pretty larger than the corresponding $\Phi(\omega_\infty^2)$ for small ω^2, while with large ω^2 the former are slightly smaller than the latter. The two curves intersect nearly at ($\omega^2 = 0.427$, $\Phi = 0.9386$).

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2) About some parts of computations, especially in regard to numerical solution of $P_{n-1}(\varsigma)=0$, the writer owes to his classmates, Nameda and others.

3) Y. Watanabe, loc. cit., p. 30.

Likewise behaves the curve $\Phi(\omega_9^2)$ to $\Phi(\omega_\infty^2)$ also, but much more approaching to it, and the point of intersection being ($\omega^2 = 0.26$, $\Phi = 0.8242$).

Prof. Watanabe indicates in the end of his note loc. cit., that to test with $\Phi(\omega_\infty^2)$ the hypothetical distribution of Japanese male stature obtained there, it shall be somewhat inadequate.

Now, if we use $\Phi(\omega_9^2)$-Table, and test the same datum, we have, indeed, for $\omega_9^2 = 0.0102$, $\Phi(\omega_9^2) = 0.0077$, so that $1-\Phi = 0.9923$ (> 0.05), while for $\omega_\infty^2 = 0.0102$, $1-\Phi(\omega_\infty^2) = 0.9993$. As it holds still alike that the hypothetical distribution is permissive, yet the degree of probability is moderately $P = 0.9923$, less than 0.9993, which is too near 1, and thus the argument becomes more plausible.